Course No (MTH-301)

Title of the course

(Mathematics)

Credit Hours

3(3-0)

Course Incharge

(Mr. Mehtab. Ahmad)

Student List

(Total Student)

37

(Boys - 22) (Girls - 15)

MTH-301 **MATHEMATICS** 3(3-0)

Objectives:

The objective of this course is to impart knowledge, logic and skills to students necessary to explore, conjecture, reason logically, and use a variety of mathematical methods to solve problems, develop self-confidence and the ability to use quantitative and spatial information in problem solving and decision making, learn to enjoy and value mathematics, to think analytically, and to understand and appreciate the role of mathematics in everyday life, be prepared for the demands of both further education and the workplace 25

Course Outline:

Sets, well known sets, operations on sets, Fundamental properties and operations of union and intersection, De Morgan's Law, Functions, types of functions, the graph of a function, Polynomial function, Algebra of polynomial function, Algebraic functions, Estimating using ratios, Arithmetic mean for grouped and ungrouped data, Matrices, types & algebra of matrices, Determinant of a square & transpose matrix, Inverse of a matrix, Determinant as a sum of products of elements, Characteristics of Binomial Theorem, Application of Binomial theorem, Limits of functions, Properties of limits of functions, Limit at infinity, Continuity of a function at a number, Limits and one sided limits, Properties of continuous functions, Continuity on an interval, Derivatives: Rates of change & slopes of tangent lines, Slope of a tangent line to a graph, The derivative of a function Basic algebraic rules for differentiation, Rules for differentiating trigonometric functions, The chain rule, Implicit differentiation, Partial Derivates of functions of two variables, Indeterminate forms 0/0 , ∞/∞, Increasing and decreasing functions, Monotone functions, critical numbers, relative extrema, First derivative test, Concavity Point of inflection and second derivative test, Absolute extrema, Indefinite Integration, Basic algebraic rules for integration, The method of substitution or change of variable, Definite integral, Basic properties of definite integral, Trapezoidal Rule, Simpson's Rule

Recommended Books:

1. Calculus with Analytic Geometry, 4th Ed, 2000, M.A Munem, D. J. Foulis, Worth Publishers, Inc
2. Calculus with Analytic Geometry, 8th Ed, 2002, George B. Thomas, Jr, Ross L. Finney, Addison-Wesley Publishing Company
3. Calculus with Analytic Geometry, 6th Ed, 2002, Dr, S.M. Yusaf, Prof Muhammad Amin, IlmiKitabKhana, Lahore Pakistan
4. Mathematical Methods, 4th Ed, 2000, Dr. S.M. Yusaf, Dr. Abdul Majeed, Prof Muhammad Amin, IlmiKitabKhana, Lahore Pakistan.

Lecture No. 1

Sets and its Types

Set :-

Set is a collection of well-defined object/element or set is a collection of distinct object/element set of Natural number.

Example :-

$$\{1, 2, 3\}, \{a, b, c\}$$

Notation set $\{\}$ is empty.

We Known set :-

=> set of natural numbers.

$$N = \{1, 2, 3, 4, \ldots\}$$

=> set of integers

$$Z = \{0, \pm 1, \pm 2, \ldots\}$$

=> Set of rational numbers

$$Q = \{p/q, \text{ where } p, q \text{ are integers and } q \neq 0\}$$

=> Set of all real numbers

$$R = Q \cup Q'$$

Real numbers :- Combination of rational and irrational numbers is called Real numbers.

Types of Sets:-

1) Empty set
2) Singleton set
3) Finite set
4) Infinite set
5) Equal sets
6) Equivalent sets
7) Universal set
8) Subset
9) proper subset
10) Superset
11) proper superset
12) power set

Universal set:-

A set containing every things is called universal set.

Example:-

$U = \{1, 2, 3, \cdots 10\}$, $A = \{1, 2, 3\}$, $B = \{4, 5, 6\}$

Subset:-

part of set is called subset.

$U = \{1, 2, 3, \cdots 10\}$

Subset

$A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$

$A \subset U$, $B \subset U$

Notation of subset is "$\subset$".

Lecture No. 2

Operation on Sets

Well-Known Set :-

Set of natural numbers:-

$N = \{1, 2, 3, \dots\}$

Set of Integers:

$Z = \{0, \pm 1, \pm 2, \dots\}$

Set of Rational numbers:

$Q = \{ p/q$, where, p, q are integers and $q \neq 0\}$

Set of all real number:

$R = Q \cup Q'$

Operation On Set:-

There are the following operation on sets.

(i) Union

(ii) Intersection

(iii) Difference of two sets.

(iv) Compliment of sets

i) Union of Sets:-

Let A and B be any two is defined as.

$A \cup B = \{x : x \in A \text{ OR } x \in B\}$

Example:-

$A = \{1, 2, 3, 4\}$, $C = \{3, 7, 8, 10\}$

$B = \{4, 5, 6, 9\}$

i) $A \cup B = \{1, 2, 3, 4\} \cup \{4, 5, 6, 9\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9\}$

(ii) $A \cup (B \cup C)$

Solve yourself

Q-NO-1

$A = \{a, b, c, d\}$

$B = \{c, d, e\}$

$C = \{g, K, l\}$

i) $A \cup (B \cup C)$ (ii) $A \cup B$ (iii) $(A \cup B) \cup C$

Properties

If $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$

$C = \{3, 4, 5, 6, 7, 8\}$, $U =$

Then prove

i) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(i) Distributive property of Union over Intersection.

(ii) Distributive property of intersection over union.

* *

(6)

$(A \cup B)' = A' \cap B'$

$L.H.S = (A \cup B)'$

$A \cup B = \{a, c\} \cup \{a, b, c, d\}$

$A \cup B = \{a, b, c, d\}$

$(A \cup B)' = U - (A \cup B) = \{a, b, c, d, e\} - \{a, b, c, d\}$

$(A \cup B)' = \{e\}$

$R.H.S = A' \cap B'$

$A' = U - A = \{a, b, c, d, e\} - \{a, c\}$

$A' = \{b, d, e\}$

$B' = U - B = \{a, b, c, d, e\} - \{a, b, c, d\}$

$B' = \{e\}$

$A' \cap B' = \{b, d, e\} \cap \{e\}$

$A' \cap B' = \{e\}$

$L.H.S = R.H.S$

Lecture NO-3

Property of Union and Intersection

Properties of Intersection

i) $A \cap A = A$

ii) $A \cap B = B \cap A$

iii) $A \cap (B \cap C) = (A \cap B) \cap C$

iv) $A \cap U = A$

v) $A \cap \phi = \phi$

=> For Example

$A = \{1, 2, 3\}$, $\phi = \{\ \}$

$A \cap \phi = \{1, 2, 3\} \cap \{\ \}$

$A \cap \phi = \{\ \}$

∵ Solve other properties by yourself.

Difference of two sets:

Let A or B be two sets then difference b/w two sets is defined as.

$A - B$ = Difference of B from A

$= \{x : x \in A \text{ and } x \notin B\}$

Properties of Union and Intersetion

Let A, B and C any three sets

properties of Union

i) $A \cup A = A$

ii) $A \cup B = B \cup A \rightarrow$ commutative

iii) $A \cup (B \cup C) = (A \cup B) \cup C$

iv) $A \cup U = U$

v) $A \cup \phi = A$

verify $A \cup (B \cup C) = (A \cup B) \cup C$ yourself

(ii) $A \cup B = B \cup A$ if $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$

L.H.S $= A \cup B = \{1, 2, 3\} \cup \{2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5\}$

R.H.S $= B \cup A = \{2, 3, 4, 5\} \cup \{1, 2, 3\}$

$B \cup A = \{1, 2, 3, 4, 5\}$

Lecture No. 4

Demorgan's Law

properties of compliment

i) $(A')' = A$

(ii) $A' \cup A = U$, $A' \cap A = \phi$

(iii) $A' \cup \phi = A'$, $A' \cap \phi = \phi$

iv) $U' = \phi$, $\phi' = U'$

=> If $A = \{1, 2, 3\}$ verify all above

$B = \{2, 3, 4, 5\}$

$U = \{1, 2, 3, \dots, 10\}$

=> **Demorgans Law**

i) $(A \cup B)' = A' \cap B'$

ii) $(A \cap B)' = A' \cup B'$.

Some important results:-

$A \subset B = A \cup B = B$

$A \cap B = A$

(i)

$A = \{a, c\}$

$B = \{a, b, c, d\}$

$U = \{a, b, c, d, e\}$

If $A = \{3, 6, 9\}$, $B = \{2, 4, 8\}$ then prove $U = \{1, 2, 3, \dots 10\}$

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

prove above

=> For Example

$$A = \{1, 2, 3, 4, 5, 6\}, \ B = \{5, 6, 7, 8, 9, 10\}$$

$$A - B = \{1, 2, 3, 4, 5, 6\} - \{5, 6, 7, 8, 9, 10\}$$

$$A - B = \{1, 2, 3, 4\}$$

$$B - A = \{7, 8, 9, 10\}$$

$B - A$ = Difference of A from B

$$= \{x : x \in B \text{ and } x \notin A\}$$

Compliment of Set:-

Let A be any set and "U" Universal then compliment of set is define as

Notation of compliment

$$A^c, A', \bar{A}$$

$$A = \{x : x \in U \text{ and } x \notin A\}$$

=> Example:-

$$U = \{1, 2, 3, \ldots, 10\}$$

$$A = \{1, 2, 3\}$$

$$A' = U - A = \{1, 2, 3, \ldots, 10\} - \{1, 2, 3\}$$

$$A' = \{4, 5, 6, 7, 8, 9, 10\}$$

(i) $A \cup (B \cup C)$

$$A \cup (B \cup C) = \{a, b, c, d\} \cup [\{c, d, e\} \cup \{g, k, l\}]$$

$$A \cup (B \cup C) = \{a, b, c, d\} \cup \{c, d, e, g, k, l\}$$

$$A \cup (B \cup C) = \{a, b, c, d, e, g, k, l\}$$

(ii) $A \cup B$ => solve yourself

(iii) $(A \cup B) \cup C$ => Do yourself

(ii) Intersetion :-

Let A and B be two any sets then intersection b/w two sets is defined as

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

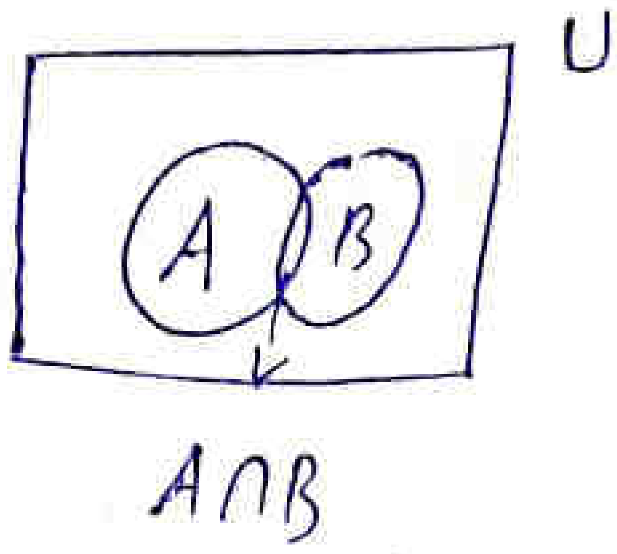
Notation of Intersection is "$\cap$" OR and

For Example:-

$$A \cap B = \{a, b\} \cap \{b, c\}$$
$$= \{b\}$$

(12)

Venn Diagram



Q. NO. 1

$A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4\}$

$C = \{3, 4\}$

(i) $A \cap (B \cap C)$

$A \cap (B \cap C) = \{1, 2, 3, 4\} \cap [\{2, 3, 4\} \cap \{3, 4\}]$

$= \{1, 2, 3, 4\} \cap \{3, 4\}$

$A \cap (B \cap C) = \{3, 4\}$

(ii) $A \cap B$

(iii) $(A \cap B) \cap C$

Solve yourself

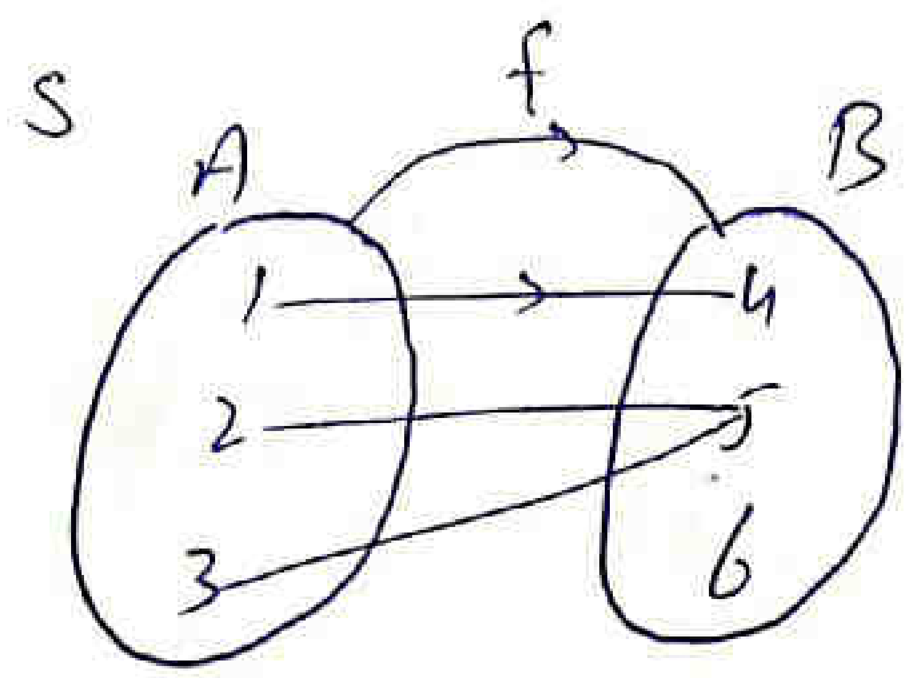
Lecture No. 5

Function

Function:

Function is a relation from set A to B in which every element of A has unique imge in B

$f: A \to B$



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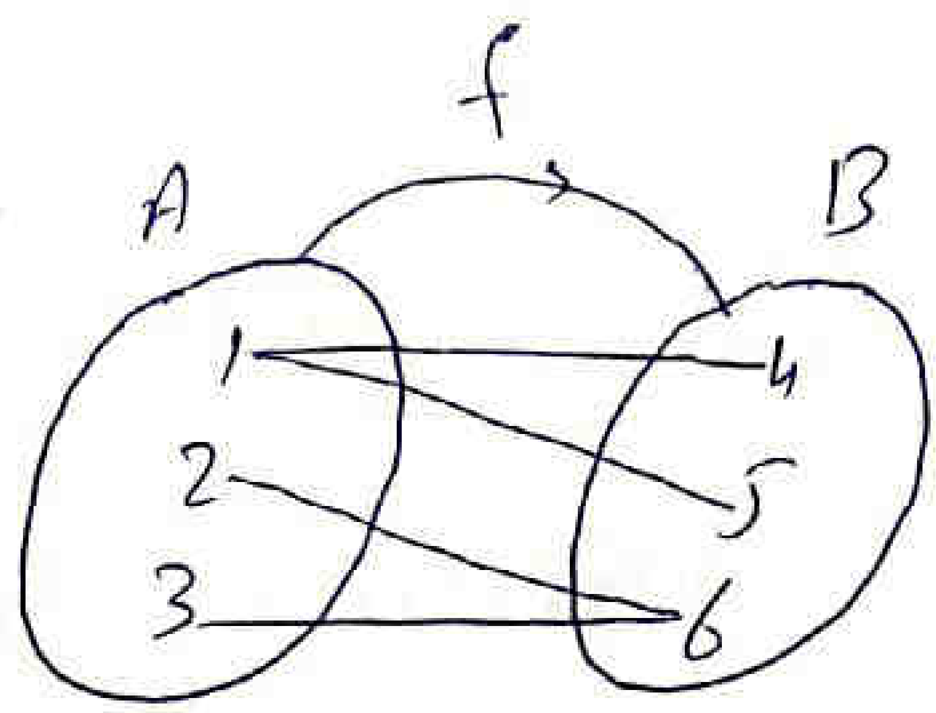
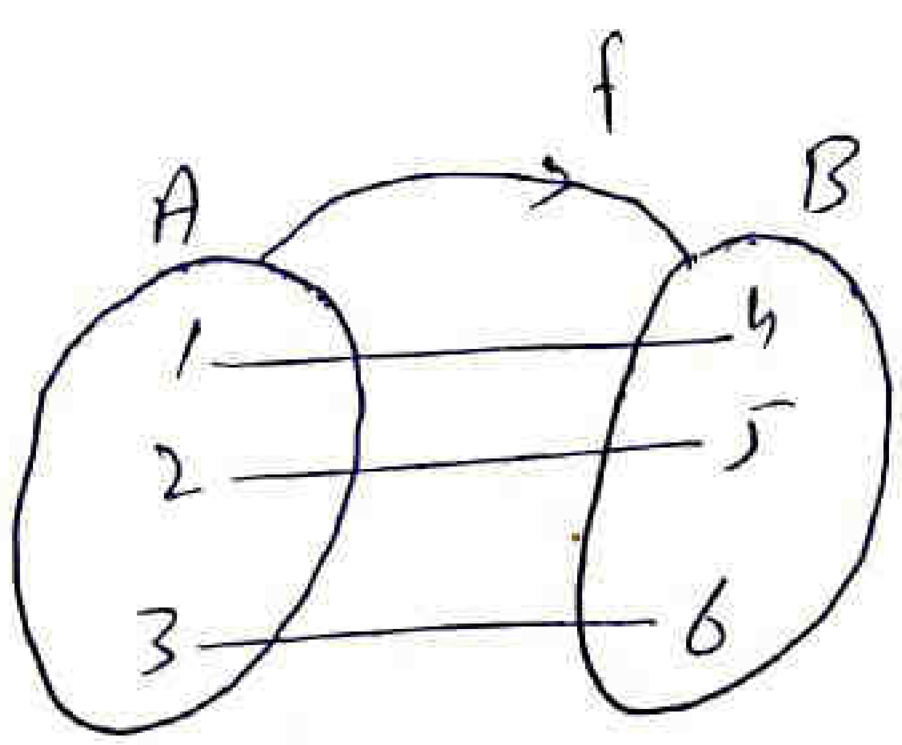


not-functio



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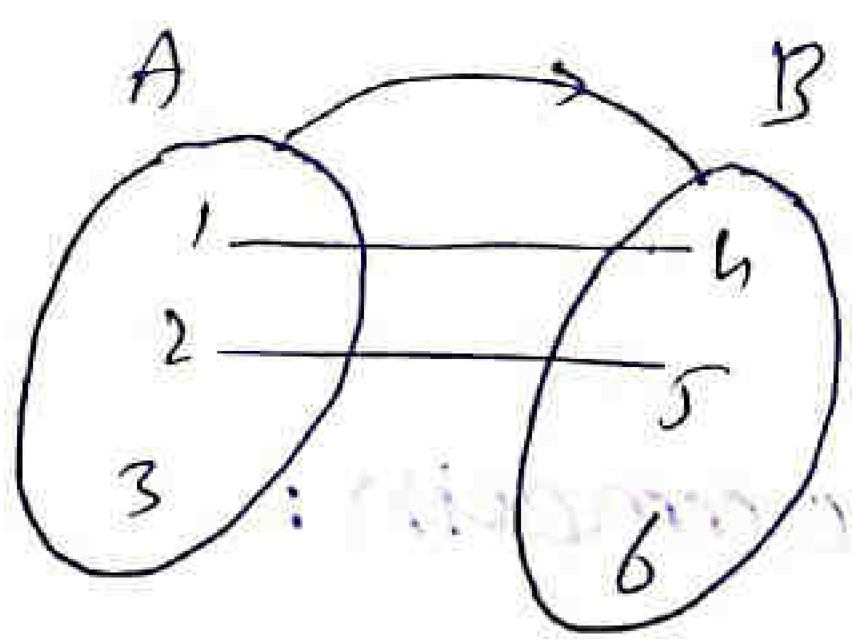
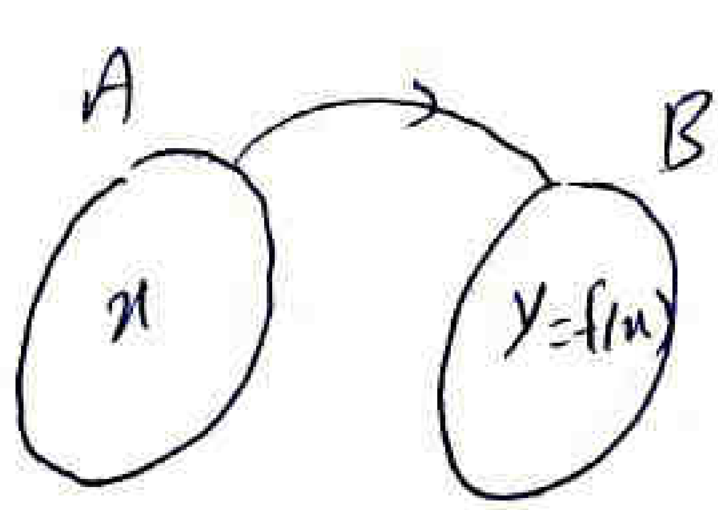


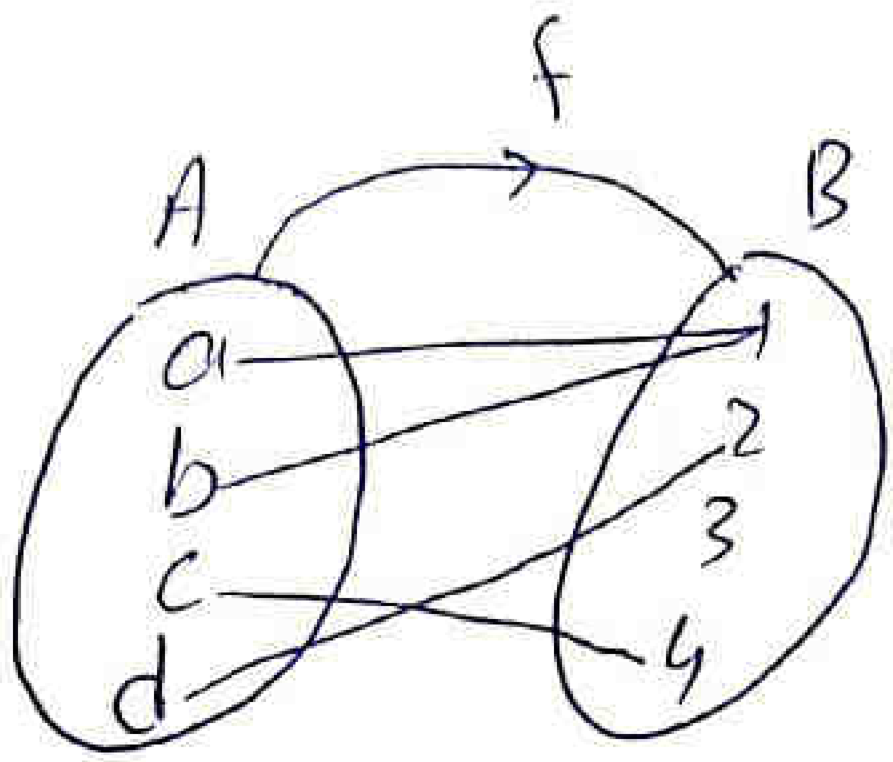
not - function



$y = f(x),\ x \in A,\ \&\ y \in B$

that is y is the imge of x

& x is the pre imge of y



$f : A \rightarrow B$ ⟶ Codomain

↓

Domain

domain of function (D_f) = $\{a, b, c, d\}$

Range of function (R_f) = $\{1, 2, 4\}$

co domain = $\{1, 2, 3, 4\}$

Domain:

all possible input value

Range:

all possible out put value

Codomain:

If $f : A \rightarrow B$ is a function from set A to B then

Codomain = B

Example :

$f : N \to N$ define

by $f(x) = 2x + 3 \to$ ①

As set of ordered pair

Find domain, Range, codomain

Sol

$N = 1, 2, 3 \ldots$

let $x = N = 1, 2, 3 \ldots$

put in ① one by one

then we get

$f = \{(1,5), (2,7), (3,9) \ldots\}$

$D_f = \{1, 2, 3 \ldots\}$

$R_f = \{5, 7, 9 \ldots\}$

codomain $= N =$ set of natural number

Example :

$f = \{(1,2), (2,3), (4,3), (5,4)\}$

$D_f = \{1, 2, 4, 5\}$

$R_f = \{2, 3, 4\}$

Lecture No. 6

Types of Function

Types of function:

There are following types of function

(i) one-one function → Injective

ii) onto-function → Subjective

bijective

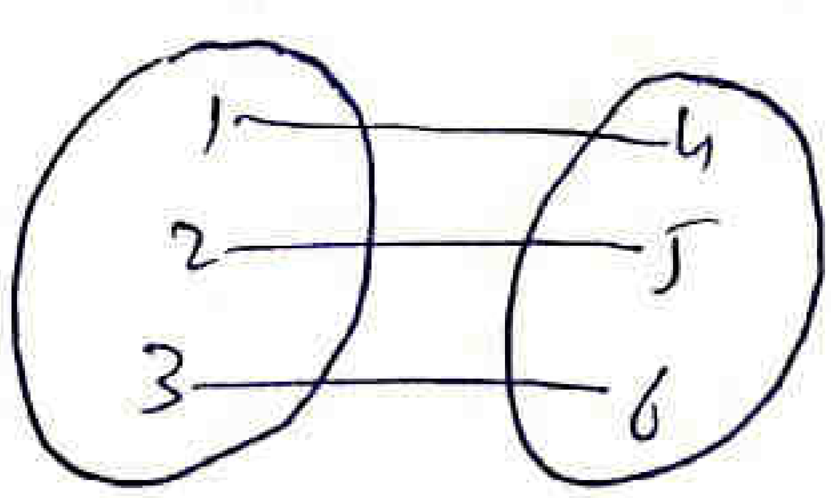
iii) into-function

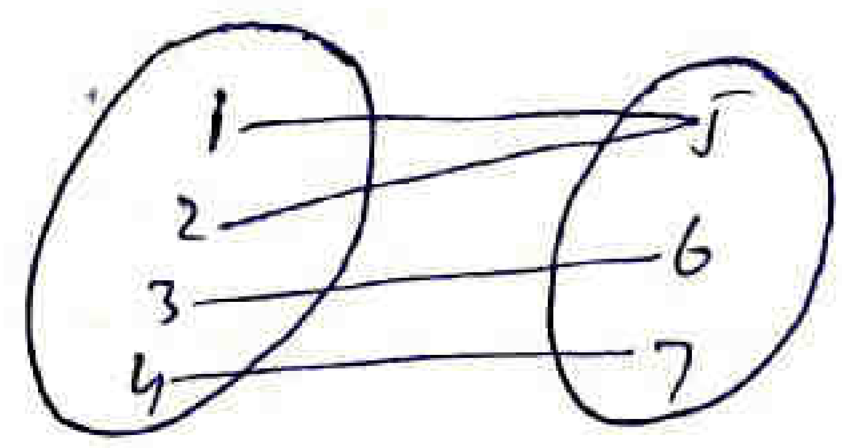
one-one Function:

A function $f: A \to B$ is one-one if for any

$f(x_1) = f(x_2) \Rightarrow x_1 = x_2$

i.e image of distinct element of A under f mapping (function) are distinct.





many-one Function

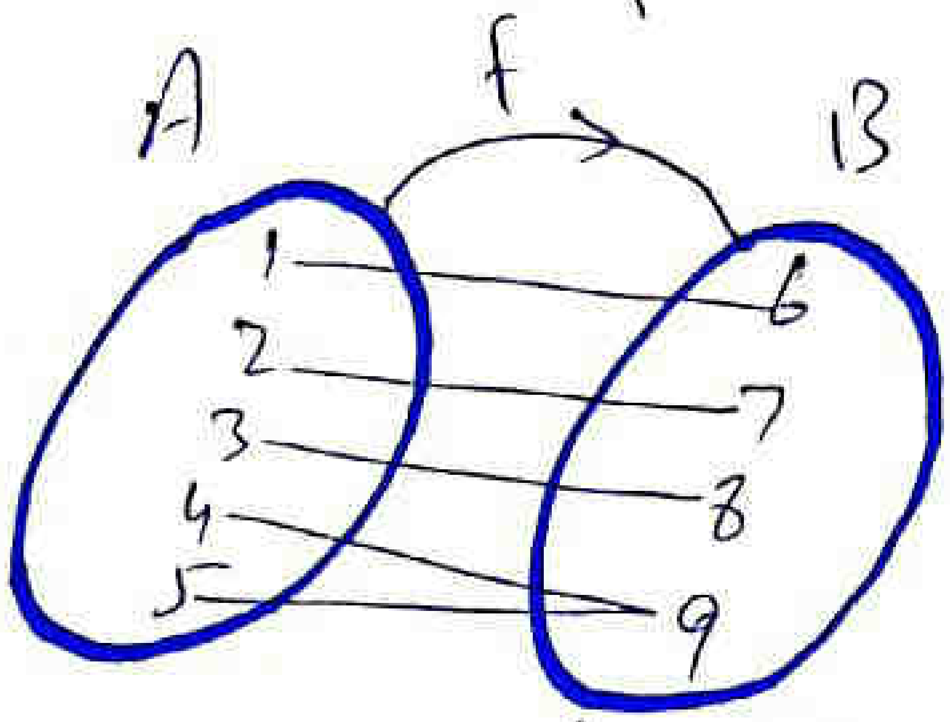
ii) onto- function:

A function $f: A \to B$ is Said to be onto if

Range of $f = B$

that is

$$f(A) = B$$

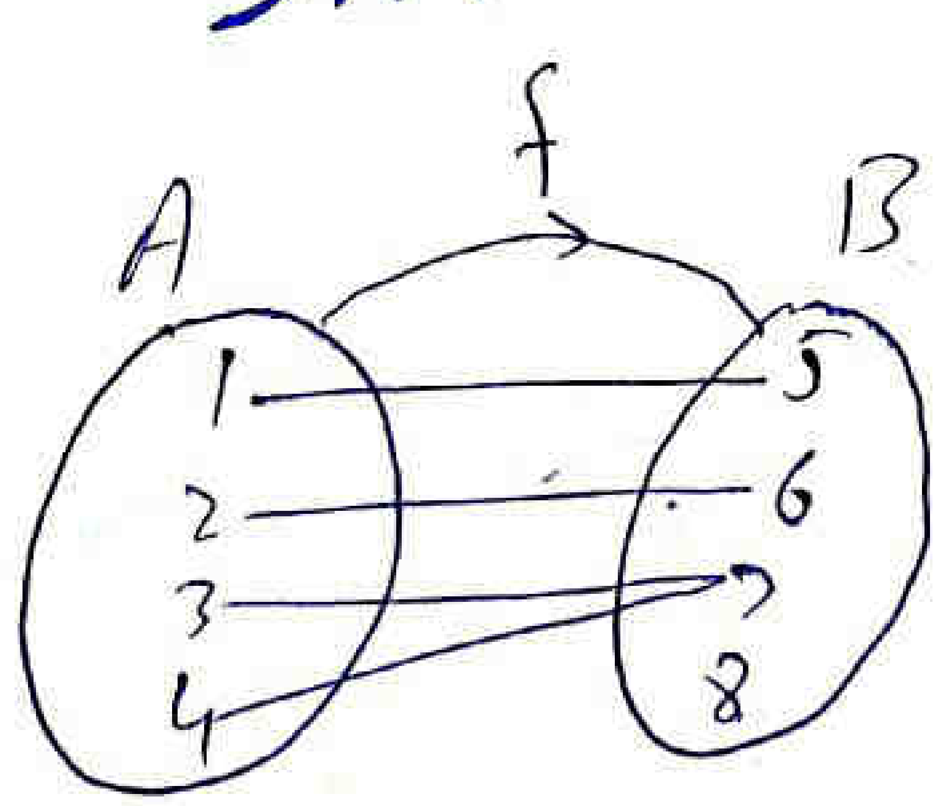


Koi bhi element chotna nhi chahia in codomain

OR

If codomain & Range are equal then function is onto

iii) Into - Function:



Lecture No. 7

Graph of Function

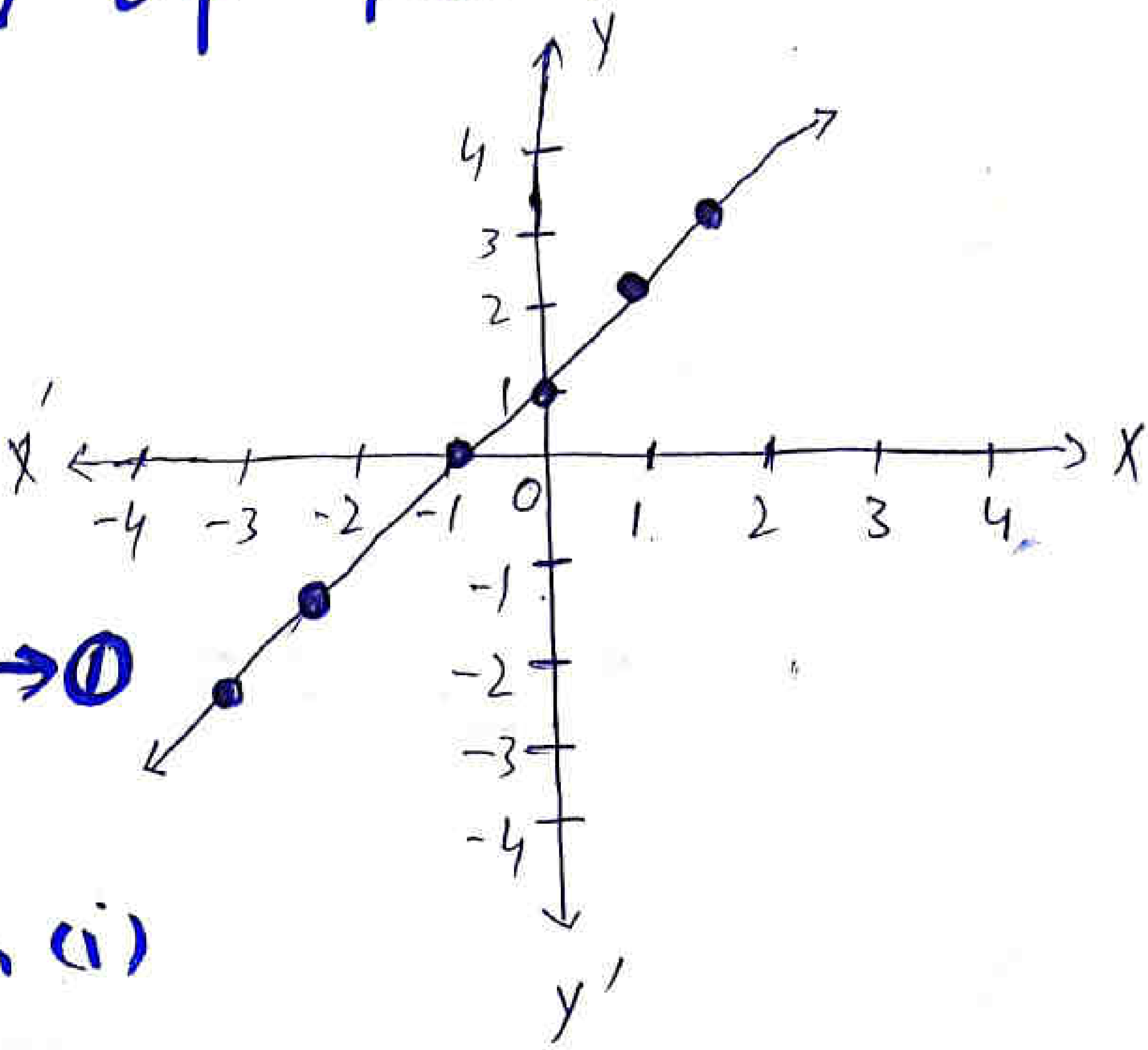
Graph of function

consider:

$$f(x) = x+1$$

Since

$$y = f(x) = x+1 \rightarrow (1)$$



Now:

put $x=0$ in (i)

$$y = f(0) = 0+1 = 1$$

$$(x,y) = (0,1)$$

put $x = 1$ in (i)

$$y = f(1) = 1+1 = 2$$

$$(x,y) = (1,2)$$

put $x = 2$

$$y = f(2) = 2+1 = 3$$

$$(x,y) = (2,3).$$

put $x = -2$ in (i)

$$y = f(-2) = -2+1 = -1$$

$$y = -1$$

$$(x,y) = (-2,-1)$$

put $x = -3$ in (i)

$$y = f(-3) = -3+1 = -2$$

$$y = -2$$

$$(x,y) = (-3,-2)$$

Now: put $x = -1$ in (i)

$$y = f(-1) = -1+1 = 0$$

$$y = 0$$

$$(x,y) = (-1,0)$$

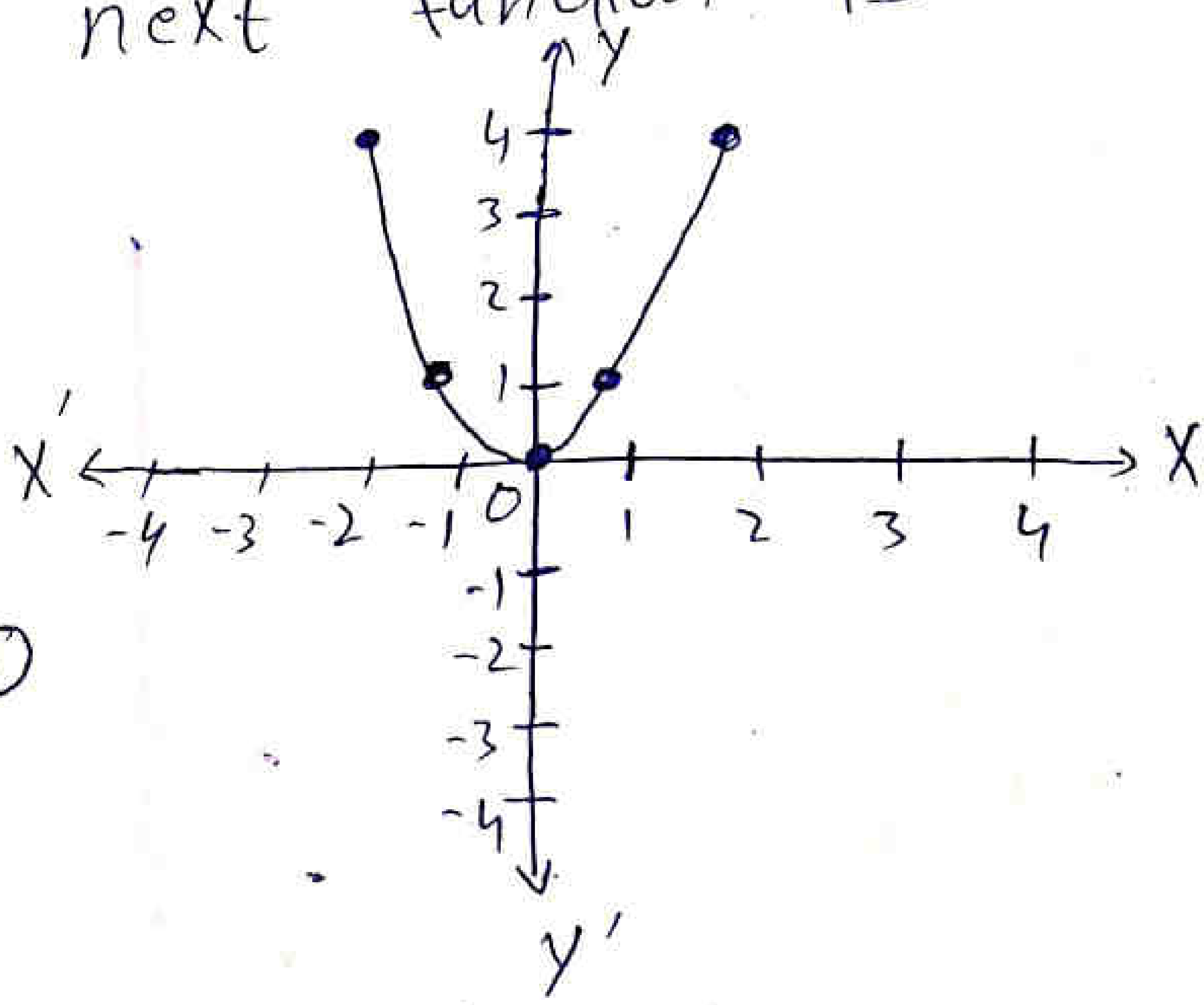
the function

$y = f(x) = x+1$

give us a graph of stright line

How: we discuss the graph of next function is

$f(x) = x^2$



Sol

Let

$y = f(x) = x^2 \rightarrow (1)$

put $x=0$ in (1)

$y = f(0) = 0^2 = 0$

$y = 0$

$(x, y) = (0,0)$

put $x=1$

$y = f(1) = 1^2 = 1$

$y = 1$

$(x,y) = (1,1)$

put $x=2$ in (1)

$y = f(2) = 2^2 = 4$

$y = 4$

$(x, y) = (2,4)$

put $x = -1$

$y = f(-1) = (-1)^2 = 1$

$(x,y) = (-1,1)$

put $x = -2$

$y = f(-2) = (-2)^2 = 4$

$(x,y) = (-2, +4)$

this graph is parabolic

Lecture No. 8

Matrix and Order of Matrix

Matrix And Determinants:

Matrix:

The arrangements of elements in a Rows And columns in a Squar brackets is called matrix

Example:

i) $\begin{bmatrix} 1 & 2 & 3 \\ 6 & 7 & 9 \end{bmatrix}$ ii) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Types of Matrix:

Rows:

The horizontal lines of numbers in a bracket are called Rows

Example

i) $\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$ ii) $\begin{bmatrix} a & b & c \end{bmatrix}$

columns:

The verticle lines of numbers in a bracket are called columns

Example:

i) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ ii) $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$

order of a matrix:

The number of rows And columns in a matrix specifies its order

If a matrix M has m rows and n columns, then M is said to be of order m-by-n OR m by n

Example:

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$$

where

number of rows = m = 2

number of columns = n = 3

So

order of matrix M = m-by-n

= 2-by-3

= 2 by 3

(23)

Find the order of Following matrix :-

i) $A = \begin{bmatrix} 2 & 3 \\ -5 & 6 \end{bmatrix}$ ii) $B = \begin{bmatrix} 2 & 0 \\ 3 & 5 \end{bmatrix}$

iii) $C = \begin{bmatrix} 2 & 4 \end{bmatrix}$ iv) $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

v) $E = \begin{bmatrix} a & d \\ b & e \\ c & f \end{bmatrix}$ vi) $H = \begin{bmatrix} 2 & 3 & 0 \\ 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$

Sol $D = \begin{bmatrix} 4 \\ 0 \\ 6 \end{bmatrix}$

number of rows $= m = 3$

number of columns $= n = 1$

order of $D = m$-by-n

$= 3$-by-1

Similarly solve other Question

Rectangular matrix:

A matrix M is called rectangular matrix if the number of rows of M is not equal to the number of columns of M

Example:

i) $A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 3 \end{bmatrix}$ ii) $D = \begin{bmatrix} 7 \\ 8 \\ 0 \end{bmatrix}$

Squar Matrix:

A matrix is called a squar matrix, if its number of rows is equal to its number of columns

Example:

i) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ ii) $B = \begin{bmatrix} 3 \end{bmatrix}$

Null oR Zero Matrix:

A matrix M is called null oR zero matrix if all elements of M is 0 e.g $M = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Lecture No. 9

Types of Matrix

Negative of a matrix

Let A be a matrix. then its negative, -A is obtained by changing the signs of all the entries.

that is

If $A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}$, then $-A = \begin{bmatrix} -1 & 2 \\ -3 & -4 \end{bmatrix}$

Symmetric Matrix:

A square matrix is symmetric, if it is equal to its transpose

that is

matrix A is symmetric if $A^t = A$

Example:

$$M = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}$$ is a square matrix

then

$$M^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix}^t$$

$$M^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 4 & 0 \end{bmatrix} = M$$

$M^t = M$. Hence M is a symmetric matrix

Transpose of Matrix:

A matrix obtained by changing the rows into columns or columns into rows of a matrix is called transpose of that matrix.

If A is a matrix, then its transpose is denoted by A^t

Example:

i) $A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \end{bmatrix}$

$A^t = \begin{bmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \end{bmatrix}^t$

$A^t = \begin{bmatrix} 1 & 2 \\ 0 & -1 \\ 2 & 3 \end{bmatrix}$

ii)

$B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 4 & -2 \end{bmatrix}$, $B^t = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 0 \\ -1 & 4 & -2 \end{bmatrix}^t$

$B^t = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & 4 \\ 3 & 0 & -2 \end{bmatrix}$

Skew-Symmetric Matrix:

A square matrix A is said to be skew-symmetric, if $A^t = -A$

Example:

if
$$A = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}, \text{ then}$$

$$A^t = \begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}^t = \begin{bmatrix} 0 & -2 & -3 \\ 2 & 0 & -1 \\ 3 & 1 & 0 \end{bmatrix}$$

$$A^t = -\begin{bmatrix} 0 & 2 & 3 \\ -2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix} = -A$$

$A^t = -A$. Hence A is a skew-symmetric matrix

Diagonal Matrix

A square matrix A is called a diagonal matrix if atleast any one of the entries of its diagonal is not zero And non-diagonal entries are zero

that is $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Scalar Matrix:

A diagonal matrix is called a scalar matrix, if all the diagonal entries are same and non-zero

Example:

i) $A = \begin{bmatrix} K & 0 & 0 \\ 0 & K & 0 \\ 0 & 0 & K \end{bmatrix}$ ii) $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

Identity Matrix:

A diagonal matrix is called identity (unit) matrix if all diagonal entries are 1.

It is denoted by I

Example:

i) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii) $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Addition And Subtraction of Matrix:

Let A And B be two Any matrix then addition And Subtraction of two matrices is defined AS

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & 7 \\ 4 & 9 \end{bmatrix}$$

Addition:

$$A+B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} 3 & 7 \\ 4 & 9 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 1+3 & 2+7 \\ 3+4 & 4+9 \end{bmatrix} = \begin{bmatrix} 4 & 9 \\ 7 & 13 \end{bmatrix}$$

Subtraction:

$$A-B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 7 \\ 4 & 9 \end{bmatrix}$$

$$A-B = \begin{bmatrix} 1-3 & 2-7 \\ 3-4 & 4-9 \end{bmatrix} = \begin{bmatrix} -2 & -5 \\ -1 & -5 \end{bmatrix}$$

Lecture No. 10

Exercise

EXERCISE :

Q : Find the determinant of the Following matrices

i) $A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$ ii) $B = \begin{bmatrix} 1 & 3 \\ 2 & -2 \end{bmatrix}$

iii) $C = \begin{bmatrix} 3 & 2 \\ 3 & 2 \end{bmatrix}$ iv) $D = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$

Sol

(i)

$A = \begin{bmatrix} -1 & 1 \\ 2 & 0 \end{bmatrix}$

$|A| = \begin{vmatrix} -1 & 1 \\ 2 & 0 \end{vmatrix} = -1 \times 0 - 2 \times 1$

$= 0 - 2 = -2$

$|A| = -2$

Similarly Solve above

Q : 2

Find the Following matrices are Singular or non-Singular

(i) $A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$ ii) $B = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$

iii) $C = \begin{bmatrix} 7 & -9 \\ 3 & 5 \end{bmatrix}$ iv) $D = \begin{bmatrix} 5 & -10 \\ -2 & 4 \end{bmatrix}$

Hint:

i) if $|A| = 0$ then matrix is Singular

ii) if $|A| \neq 0$ then matrix is non-Singular

Hint:
i) If determinant$(A) = 0$ then Singular
ii) If determinant $(A) \neq 0$ then matrix is non-Sing

Sol (i)

$$A = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 3 & 6 \\ 2 & 4 \end{vmatrix} = 3 \times 4 - 2 \times 6$$

$$|A| = 12 - 12 = 0$$

$$|A| = 0$$

Hence matrix A is Singular matrix

Similarly Solve above

Q: 3

Find the inverse of Following matrix:

i) $A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$ ii) $B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}$

iii) $C = \begin{bmatrix} -2 & 6 \\ 3 & -9 \end{bmatrix}$ iv) $D = \begin{bmatrix} \frac{1}{2} & \frac{3}{4} \\ 1 & 2 \end{bmatrix}$

Sol (i)

Since

$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}$

we know that inverse of matrix A define AS

$A^{-1} = \frac{1}{|A|}(Adj A) \rightarrow (1)$

Now:

$|A| = \begin{vmatrix} -1 & 3 \\ 2 & 0 \end{vmatrix} = 0 \times (-1) - 2 \times 3 = 0 - 6 = -6$

$|A| = -6 \neq 0$

And

$(Adj A) = \begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix}$

So Equation ① become.

$$A^{-1} = \frac{1}{|A|}(Adj\,A).$$

$$A^{-1} = -\frac{1}{6}\begin{bmatrix} 0 & -3 \\ -2 & -1 \end{bmatrix} = \begin{bmatrix} 0 \times -\frac{1}{6} & -3 \times (-\frac{1}{6}) \\ -2 \times (-\frac{1}{6}) & -1 \times (-\frac{1}{6}) \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{3}{6} \\ \frac{2}{6} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\cancel{3}}{2 \times \cancel{3}} \\ \frac{\cancel{2}}{3 \times \cancel{2}} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\cancel{3}^{1}}{2 \times \cancel{3}} \\ \frac{\cancel{2}^{1}}{3 \times \cancel{2}} & \frac{1}{6} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

Similarly solve above all

Q: 4

If $A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$ then

i) $A(Adj\,A) = (Adj\,A)A = (\det A)I$

ii) $BB^{-1} = I = B^{-1}B$

Since $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

↓

Identity matrix

Sol (i)

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$A(\text{Adj} A) = (\text{Adj} A) A = (\det A) I$$

Take part $A(\text{Adj} A)$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}, \quad (\text{Adj} A) = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$\Rightarrow$

$$A(\text{Adj} A) = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix} \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times 6 + 2\times(-4) & 1\times(-2) + 2\times 1 \\ 4\times 6 + 6\times(-4) & 4\times(-2) + 6\times 1 \end{bmatrix}$$

$$A(\text{Adj} A) = \begin{bmatrix} 6-8 & -2+2 \\ 24-24 & -8+6 \end{bmatrix}$$

$$A(\text{Adj} A) = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow (1)$$

Now:

Take

$$(\text{Adj} A) A = \begin{bmatrix} 6 & -2 \\ -4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 6\times 1 + (-2)\times 4 & 6\times 2 + (-2)\times 6 \\ -4\times 1 + 1\times 4 & -4\times 2 + 1\times 6 \end{bmatrix} = \begin{bmatrix} 6-8 & 12-12 \\ -4+4 & -8+6 \end{bmatrix}$$

$$(Adj A)A = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow (ii)$$

Now:

Take part

$(\det A) I$

$$A = \begin{bmatrix} 1 & 2 \\ 4 & 6 \end{bmatrix}$$

$$\det A = |A| = \begin{vmatrix} 1 & 2 \\ 4 & 6 \end{vmatrix} = 1 \times 6 - 4 \times 2 = 6 - 8 = -2$$

$(\det A) = -2$,

we know that

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Since

$$(\det A) I = -2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \rightarrow (iii)$$

See Equations. (i), (ii) & (iii)

Hence prove

$$A(Adj A) = (Adj A)A = (\det A) I$$

prove above part

that is

$$BB^{-1} = B^{-1}B = I$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 3 & -1 \\ 2 & -2 \end{bmatrix}$$

Lecture No. 11

Adjoint and Inverse of Matrix

Adjoint of a Matrix:

Adjoint of a square matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is obtained by interchanging the diagonal entries and changing the sign of other entries. Adjoint of matrix is denoted by

$$\therefore \text{Adj } A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Example:

i) $B = \begin{bmatrix} 2 & 6 \\ 7 & 0 \end{bmatrix}$

$$(\text{Adj } B) = \begin{bmatrix} 0 & -6 \\ -7 & 2 \end{bmatrix}$$

ii) $C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

$$(\text{Adj } C) = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

Inverse of a Matrix

Inverse of a matrix is define

As

$$A^{-1} = \frac{1}{|A|}(\text{Adj}\,A) = \frac{(\text{Adj}\,A)}{|A|}$$

OR

$$A^{-1} = \frac{(\text{Adj}\,A)}{\det(A)}$$

Example:

Find A^{-1} if $A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$ And

verify that $AA^{-1} = A^{-1}A$

Sol

we know that

$$A^{-1} = \frac{(\text{Adj}\,A)}{|A|} \rightarrow \text{(A)}$$

Now

Since $A^{-1} = \frac{(Adj\,A)}{|A|} \rightarrow$ A

$$A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 3 \\ 1 & 1 \end{vmatrix}$$

$$|A| = 5\times1 - 3\times1 = 5 - 3 = 2$$

$$|A| = 2 \neq 0$$

Now

$$(Adj\,A) = \begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$$

so A become

$$A^{-1} = \frac{(Adj\,A)}{|A|} = \frac{1}{|A|} \cdot (Adj\,A)$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{2}\begin{bmatrix} 1 & -3 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2}\times1 & -3\times\frac{1}{2} \\ -1\times\frac{1}{2} & 5\times\frac{1}{2} \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & \frac{-3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

Now:

prove

$$AA^{-1} = A^{-1}A$$

Since

$$A^{-1} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

$$A = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$$

Sol

Take L.H.S

$$AA^{-1} = \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 5 \times \frac{1}{2} + 3 \times (-\frac{1}{2}) & 5 \times (-\frac{3}{2}) + 3 \times \frac{5}{2} \\ 1 \times (+\frac{1}{2}) + 1 \times (-\frac{1}{2}) & 1 \times (-\frac{3}{2}) + 1 \times \frac{5}{2} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} \frac{5}{2} - \frac{3}{2} & -\frac{15}{2} + \frac{15}{2} \\ \frac{1}{2} - \frac{1}{2} & -\frac{3}{2} + \frac{5}{2} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} \frac{5-3}{2} & 0 \\ 0 & \frac{-3+5}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{2} & 0 \\ 0 & \frac{2}{2} \end{bmatrix}$$

$$AA^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow (1)$$

Now:

Take R.H.S

$$A^{-1}A = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix} = \Big[\frac{1}{2} \times 5 + (-\frac{3}{2}) \times 1$$

Now:

Take R.H.S

$$A^{-1}A = \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ -\frac{1}{2} & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 5 & 3 \\ 1 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{1}{2} \times 5 + \left(-\frac{3}{2}\right) \times 1 & \frac{1}{2} \times 3 + \left(-\frac{3}{2}\right) \times 1 \\ -\frac{1}{2} \times 5 + \frac{5}{2} \times 1 & -\frac{1}{2} \times 3 + \frac{5}{2} \times 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{5}{2} - \frac{3}{2} & \frac{3}{2} - \frac{3}{2} \\ -\frac{5}{2} + \frac{5}{2} & -\frac{3}{2} + \frac{5}{2} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{5-3}{2} & \frac{3-3}{2} \\ \frac{-5+5}{2} & \frac{-3+5}{2} \end{bmatrix} = \begin{bmatrix} \frac{2}{2} & 0 \\ 0 & \frac{2}{2} \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \rightarrow (11)$$

See from Eq (1) & (11)

L.H.S = R.H.S

Hence prove

$$AA^{-1} = A^{-1}A$$

Lecture No. 12

Determinat of 2-by-2 Matrix

Determinant of 2-by-2 Matrix

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ be a 2-by-2 square matrix. the determinant of A denoted by det A or $|A|$ is define AS

$$|A| = \det A = \det \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

$$= a \times d - b \times c =$$

$$= ad - bc$$

oR

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Example:

$$B = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

find determinant

$$|B| = \begin{vmatrix} 1 & 1 \\ -2 & 3 \end{vmatrix} = 1 \times 3 - 1 \times (-2)$$

$$= 3 + 2 = 5$$

Singular And Non-Singular Matrix:

=>

Singular Matrix:

A Square matrix A is called Singular if the determinant of A is equal to zero

that is

$|A| = 0$

Example

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = 1 \times 0 - 0 \times 2 = 0 - 0 = 0$$

$|A| = 0$

Non-Singular Matrix:

A Square matrix A is called Singular if the determinant of A is not Equal to zero: **Example**

$$|A| = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = 1 \times 4 - 2 \times 3 = 4 - 6 = -2 \neq 0$$

$|A| \neq 0$

Lecture No. 13

Exercise

Q: 5

$$A=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix},\quad B=\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix},\quad D=\begin{bmatrix}3 & 1\\ -2 & 2\end{bmatrix}$$

then verify that:

i) $(AB)^{-1}=B^{-1}A^{-1}$ ii) $(DA)^{-1}=A^{-1}D^{-1}$

Sol (i)

$$(AB)^{-1}=B^{-1}A^{-1}$$

Take L.H.S

$(AB)^{-1}$

Now find first $AB=?$

$$AB=\begin{bmatrix}4 & 0\\ -1 & 2\end{bmatrix}\begin{bmatrix}-4 & -2\\ 1 & -1\end{bmatrix}=\begin{bmatrix}4\times(-4)+0\times1 & 4\times(-2)+0\times(-1)\\ -1\times(-4)+2\times1 & -1\times(-2)+2\times(-1)\end{bmatrix}$$

$$AB=\begin{bmatrix}-16+0 & -8+0\\ 4+2 & 2-2\end{bmatrix}=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$$

$$AB=\begin{bmatrix}-16 & -8\\ 6 & 0\end{bmatrix}$$

$$(\text{Adj } AB)=\begin{bmatrix}0 & 8\\ -6 & -16\end{bmatrix},\quad |AB|=\begin{vmatrix}0 & 8\\ -6 & -16\end{vmatrix}$$

$$=0\times(-16)+6\times8=48$$

Since

$$(AB)^{-1} = \frac{1}{|AB|}(\text{Adj } AB) \rightarrow (I)$$

$$, AB = \begin{bmatrix} -16 & -8 \\ 6 & 0 \end{bmatrix}$$

Now

$$(\text{Adj } AB) = \begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix}$$

$$|AB| = \begin{vmatrix} -16 & -8 \\ 6 & 0 \end{vmatrix} = 0 \times (-16) - 6 \times (-8)$$

$$|AB| = -16 \times 0 - 6 \times (-8) = 0 + 48 = 48$$

$$= 0 - 6 \times (-8) = 48$$

$$|AB| = 48 \neq 0$$

So Eq (I) become

$$(AB)^{-1} = \frac{1}{|AB|}(\text{Adj } AB)$$

$$= \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix} \rightarrow (i)$$

Take R. H. S

$B^{-1}A^{-1}$

Now:

Since

$$B^{-1} = \frac{1}{|B|}(\text{Adj } B)$$

$$B^{-1} = \frac{1}{|B|}(Adj\ B) \rightarrow (A)$$

Since

$$B = \begin{bmatrix} -4 & -2 \\ 1 & -1 \end{bmatrix}$$

$$|B| = \begin{vmatrix} -4 & -2 \\ 1 & -1 \end{vmatrix} = -4 \times (-1) - (-2) \times 1$$

$$|B| = 4 + 2 = 6 \neq 0$$

$$|B| = 6 \neq 0$$

$$(Adj\ B) = \begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

Now: puting value in Eq (A)

$$B^{-1} = \frac{1}{|B|}(Adj\ B)$$

$$B^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}$$

Now $A^{-1} = ?$, $A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$

Since

$$A^{-1} = \frac{1}{|A|}(Adj\ A) \rightarrow (B)$$

$$|A| = \begin{vmatrix} 4 & 0 \\ -1 & 2 \end{vmatrix} = 4 \times 2 - (-1) \times 0$$

$$= 8 + 0 = 8$$

$$|A| = 8 \neq 0$$

$$A = \begin{bmatrix} 4 & 0 \\ -1 & 2 \end{bmatrix}$$

$$(Adj\,A) = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

puting values in Eq (B)

$$A^{-1} = \frac{1}{|A|}(Adj\,A)$$

$$A^{-1} = \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

Since R.H.S

$$B^{-1}A^{-1} = \frac{1}{6}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix} \cdot \frac{1}{8}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{6 \times 8}\begin{bmatrix} -1 & 2 \\ -1 & -4 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{48}\begin{bmatrix} -1 \times 2 + 2 \times 1 & -1 \times 0 + 2 \times 4 \\ -1 \times 2 + (-4) \times 1 & -1 \times 0 + (-4) \times 4 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{48}\begin{bmatrix} -2+2 & 0+8 \\ -2-4 & +0-16 \end{bmatrix}$$

$$B^{-1}A^{-1} = \frac{1}{48}\begin{bmatrix} 0 & 8 \\ -6 & -16 \end{bmatrix} \rightarrow \text{(ii)}$$

See Eq (i) & (ii)

L.H.S = R.H.S

Hence prove

$$(AB)^{-1} = B^{-1}A^{-1}$$

Similarly prove 2nd part

$(DA)^{-1} = A^{-1}D^{-1}$

$A = \begin{pmatrix} 4 & 0 \\ -1 & 2 \end{pmatrix}, \quad D = \begin{pmatrix} 3 & 1 \\ -2 & 2 \end{pmatrix}$

Lecture No. 14

Exercise

Q : 1

If

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

then Find

i) AB ii) BA

Sol
=

(i) AB

$$AB = \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 3 \times 6 + 0 \times 5 \\ -1 \times 6 + 2 \times 5 \end{bmatrix}$$

$$AB = \begin{bmatrix} 18 + 0 \\ -6 + 10 \end{bmatrix}$$

$$AB = \begin{bmatrix} 18 \\ 4 \end{bmatrix}$$

Solve above (ii)

Q: 2 (52)

$$A = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

then verify

i) $AB = BA$ ii) $A(BC) = (AB)C$

iii) $A(B+C) = AB + AC$ iv) $A(B-C) = AB - AC$

Sol (ii)

$$A(BC) = (AB)C$$

Take L.H.S

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\left(\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}\right)$$

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1\times2+2\times1 & 1\times1+2\times3 \\ -3\times2+(-5)\times1 & -3\times1+(-5)\times3 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 2+2 & 1+6 \\ -6-5 & -3-15 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 4 & 7 \\ -11 & -18 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times4+3\times(-11) & -1\times7+3\times(-18) \\ 2\times4+0\times(-11) & 2\times7+0\times(-18) \end{bmatrix} = \begin{bmatrix} -4-33 & -7-54 \\ 8+0 & 14+0 \end{bmatrix}$$

$$A(BC) = \begin{bmatrix} -4-33 & -7-54 \\ 8 & 14 \end{bmatrix}$$ (53)

$$A(BC) = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \rightarrow (I)$$

Take R.H.S

$$(AB)C = \left(\begin{bmatrix} -1 & 3 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -3 & -5 \end{bmatrix}\right)\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -1\times1+3\times(-3) & -1\times2+3\times(-5) \\ 2\times1+0\times(-3) & 2\times2+0\times(-5) \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -1-9 & -2-15 \\ 2+0 & 4+0 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10 & -17 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -10\times2+(-17)\times1 & -10\times1+(-17)\times3 \\ 2\times2+4\times1 & 2\times1+4\times3 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -20-17 & -10-51 \\ 4+4 & 2+12 \end{bmatrix}$$

$$(AB)C = \begin{bmatrix} -37 & -61 \\ 8 & 14 \end{bmatrix} \rightarrow (II)$$

From Eq (I) & (II)

L.H.S = R.H.S

Hence prove (54)

$A(BC) = (AB)C$

Similarly prove above all

Lecture No. 15

Exercise 3.1

Exercise 3.1

Q: 1 Do yourself

Q: 4 Do yourself

Q: 2

If $A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}$, Show that $A^4 = I_2$

Sol

Take L.H.S A^4

Now:

$$A^2 = A \cdot A = \begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix}\begin{bmatrix} i & 0 \\ 1 & -i \end{bmatrix} = \begin{bmatrix} i \times i + 0 \times 1 & i \times 0 + 0 \times (-i) \\ 1 \times i + (-i) \times 1 & 1 \times 0 + (-i)(-i) \end{bmatrix}$$

$$A^2 = \begin{bmatrix} i^2 + 0 & 0 + 0 \\ i - i & 0 + i^2 \end{bmatrix} = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

Since

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix} \cdot \begin{bmatrix} i^2 & 0 \\ 0 & i^2 \end{bmatrix}$$

Since: $i^2 = -1$

(56)

$$A^4 = A^2 \cdot A^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} -1\times(-1)+0\times 0 & -1\times 0+0\times(-1) \\ 0\times(-1)+(-1)\times 0 & 0\times 0+(-1)\times(-1) \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 1+0 & 0+0 \\ 0+0 & 0+1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

which is R.H.S

$$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

Since

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Hence prove

$$A^4 = I_2$$

Q : 3 Find x And y iF

i) $$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 1 \end{bmatrix}$$

ii) $$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} y & 1 \\ -3 & 2x \end{bmatrix}$$

$x, y = ?$ (57)

Sol (i)

$$\begin{bmatrix} x+3 & 1 \\ -3 & 3y-4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -3 & 2 \end{bmatrix}$$

Since

$x+3 = 2 \rightarrow$ (I)

$3y-4 = 2 \rightarrow$ (II)

From (I)

$x+3 = 2$

$x = 2-3 = -1$

$\boxed{x = -1}$

Do yourself 3 (ii)

Now: From (II)

$3y-4 = 2$

$3y = 2+4 = 6$

$3y = 6$

$y = \frac{6}{3} = \frac{2 \times \cancel{3}}{\cancel{3}}$

$\boxed{y = 2}$

So

$x = -1, y = 2$

Q: No. 5 (58)

Do yourself

Q: 8:

If

$$A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}, \quad A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

then find a, b

Sol

Since

$$A^2 = A \cdot A = \begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix}\begin{bmatrix} 1 & 2 \\ a & b \end{bmatrix} = \begin{bmatrix} 1\times1+2\times a & 1\times2+2\times b \\ a\times1+b\times a & a\times2+b\times b \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} \rightarrow A$$

Since $A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

So A become

$$\begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1+2a & 2+2b \\ a+ab & 2a+b^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ (59)

Equality of matrices

$1+2a = 0 \rightarrow$ ①

$2+2b = 0 \rightarrow$ ⑪

① =>

$1+2a=0$

$2a = 0-1$

$2a = -1$

$\boxed{a = \frac{-1}{2}}$

⑪ =>

$2+2b=0$

$2b = 0-2$

$2b = -2$

$b = \frac{-\cancel{2}^{1}}{\cancel{2}} = -1$

$\boxed{b=-1}$

So

$a = \frac{-1}{2}$

And

$b = -1$

Q: 9

Do yourself

Q: 10

Do yourself

Q: 11

Do yourself

Exercise 3.2

Q: 2

Find the inverse of a matrices

i) $\begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$ ii) $\begin{bmatrix} -2 & 3 \\ -4 & 5 \end{bmatrix}$

iii) $\begin{bmatrix} 2i & i \\ i & -i \end{bmatrix}$ iv) $\begin{bmatrix} 2 & 1 \\ 6 & 3 \end{bmatrix}$

Sol (i)

let

$$A = \begin{bmatrix} 3 & -1 \\ 2 & 1 \end{bmatrix}$$

Since

$$A^{-1} = \frac{1}{|A|}(\text{Adj } A) \rightarrow (\text{I})$$

Now

$$|A| = \begin{vmatrix} 3 & -1 \\ 2 & 1 \end{vmatrix} = 3\times 1 - 2\times(-1) = 3+2 = 5 \neq 0$$

$$(\text{Adj } A) = \begin{bmatrix} 1 & 1 \\ -2 & 3 \end{bmatrix}$$

Lecture No. 16

Complex Number

complex number :

The number of the form $x+iy$, where $x, y \in R$ And $i = \sqrt{-1}$ are called Complex number And here x is called real part. And y is called imaginary part of the complex number.

Example :

$3+4i$, $2-\frac{5}{7}i$ etc. are complex numbers

Question:

$2+i = 2+1\cdot i$

i) $3+9i$ ii) $2+i$

seperate real and imaginary part

Sol (1)

3 is a real part And 9 is imaginary part of the Complex number

ii) Sol

2 is a real part And 1 is a imaginary part

(62)

Powers of i

$i^2 = -1$

$i^3 = i^2 \cdot i = (-1) \cdot i = -i$

$i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$

Since

$i^2 = -1$

$\sqrt{(\;)} = (\;)^{1/2}$

$i = \sqrt{-1}$

$i = (-1)^{1/2}$

Simplify the following

i) i^9 ii) i^{14} iii) $(-1)^{19}$ iv) $(-1)^{-\frac{21}{2}}$

Sol (i)

$i^9 = i^8 \cdot i = (i^2)^4 \cdot i$

$= (-1)^4 \cdot i$

$= 1 \cdot i = i$

iv) $(-1)^{-\frac{21}{2}}$

Sol

$(-1)^{-\frac{21}{2}} = \dfrac{1}{(-1)^{\frac{21}{2}}} = \dfrac{1}{(-1)^{21 \times \frac{1}{2}}} = \dfrac{1}{\left((-1)^{\frac{1}{2}}\right)^{21}} = \dfrac{1}{i^{21}}$

$= \dfrac{1}{i^{20} \cdot i} = \dfrac{1}{(i^2)^{10} \cdot i} = \dfrac{1}{(-1)^{10} \cdot i} = \dfrac{1}{1 \cdot i} = \dfrac{1}{i} \times \dfrac{i}{i}$

$= \dfrac{i}{i^2} = \dfrac{i}{-1} = -i$

Multiplication - Addition Law:

$\forall\ a, b \in R,\ ab = ba$

$a(b+c) = ab + ac$

$(a+b)c = ac + bc$

properties of Equality:

Equality of numbers denoted by "=" possesse the following properties

i) **Reflexive property** $\forall a \in R,\ a = a$

ii) **Symmetric property** $\forall a, b \in R,\ a = b \Rightarrow b = a$

iii) Transitive property $\forall a, b, c \in R,\ a = b$ And $b = c$ And $a = c$

iv) Additive property: $\forall a, b, c \in R,\ a = b \Rightarrow a + c = b + c$

v) Multiplicative property $\forall a, b, c \in R$

$a = b \Rightarrow ac = bc$

And $ca = cb$

vi) **Cancellation property for addition:**

$\forall a, b, c \in R,\ a + c = b + c$

$\Rightarrow a = b$

vii)

vii) cancellation property for multiplication:

$\forall\, a, b, c \in R,\ ac = bc \Rightarrow a = b,\ c \neq 0$

properties of Inequalities

i) Trichotomy property

$\forall\, a, b \in R$

either $a = b$ or $a > b$ or $a < b$

Here $>$ = greater then ; $<$ = Less than

ii) Transitive property $\forall\, a, b, c \in R$

i) $a > b$ And $b > c \Rightarrow a > c$

ii) $a < b \wedge b < c \Rightarrow a < c$

iii) Additive property:

a)

i) $a > b \Rightarrow a + c > b + c$

ii) $a < b \Rightarrow a + c < b + c$

b)

i) $a > b$ And $c > d \Rightarrow a + c > b + d$

ii) [redacted]

$a < b$ And $c < d \Rightarrow a + c < b + d$

Lecture No. 17

Addition and Multiplication Law

Rational number And Irrational number

Rational number :-

Rational number is a number which can be put in the form $P/2$, where $P, 2 \in Z$, And $2 \neq 0$.

the number $\sqrt{16}$, 3, 7, 4 etc are rational numbers.

Irrational numbers:

Irrational number is a number which can not be put in the form $P/2$, where, $P, 2 \in Z$, And $2 \neq 0$

the number $\sqrt{2}$, $\sqrt{3}$, $\frac{7}{\sqrt{5}}$ are Irrational number

Addition Laws:

i) Closure Law of Addition

$\forall\, a, b \in R,\ a+b \in R$

ii) Associative Law of addition

$\forall\, a, b, c \in R,\ a+(b+c) = (a+b)+c$

iii) Additive Identity

$\forall\, a \in R,\ \exists\ 0 \in R$

Such that $a+0 = 0+a = a$

iv) Additive Inverse

v) Commutative Law for Addition

$\forall\, a \in R, b \in R$ And $a+b \in R$ then $a+b = b+a$

vii) [redacted]

Multiplication Laws:

(i) Closure Law of multiplication

$\forall =$ For all

$\forall\, a, b \in R$, $a.b \in R$

Here, R is a real number

Real number:

The Combination of rational And Irrational number is called real number.

(ii) Associative Law for multiplication

$\forall\, a, b, c \in R$, $a(bc) = (ab)c$

(iii) Multiplicative Identity

$\forall\, a \in R$, $\exists\, 1 \in R$ Such that

$\exists =$ there must exist

$a.1 = 1.a = a$

1 is the multiplicative identity of real numbers

$a^{-1}.a = \frac{1}{a}.a = \frac{a}{a} = 1$

iv) Multiple Inverse

$\forall\, a (\neq 0) \in R$, $\exists\, a^{-1} \in R$. Such that. $a.a^{-1} = a^{-1}.a = 1$

v) Commutative Law of multiplication:

$\forall\, a, b \in R$, $ab = ba$

prove following result:

i) $\frac{1}{a} \cdot \frac{1}{b} = \frac{1}{ab}$

ii) $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

iv) $\frac{a}{b} = \frac{ka}{kb}$

iv) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

v) $ab = 0 \Rightarrow a = 0$ or $b = 0$

Sol

iv)

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

Let

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{\frac{a}{b}}{\frac{c}{d}} \times \frac{bd}{bd} = \frac{\left(\frac{a}{\cancel{b}}\right)(\cancel{b}d)}{\left(\frac{c}{\cancel{d}}\right)(b\cancel{d})} = \frac{ad}{bc}$$

Expl 4:

prove that for any real numbers a, b

i) $a \cdot 0 = 0$ ii) $ab = 0 \Rightarrow a = 0$ OR $b = 0$

Solution:

$$a \cdot 0 = a\left[1 + (-1)\right]$$
$$= a(1-1)$$
$$= a \cdot 1 - a \cdot 1$$
$$= 0.$$

thus
$$a \cdot 0 = 0$$

Exple: 5:

For real numbers a, b then prove following results:

i) $(-a)(-b) = ab$ ii) $(-a)b = a(-b) = -ab$

Sol

i)

Let

$$(-a)(-b) - ab = (-a)(-b) + (-ab)$$
$$= (-a)(-b+b)$$
$$= (-a) \cdot 0$$
$$(-a)(-b) - ab = 0$$

$(-a)(-b) = 0 + ab$

$(-a)(-b) = ab$

Hence prove.

Similarly prove next result

i-e

$(-a)b = a(-b) = -ab$

Example. No. 6:

1) $\frac{a}{b} = \frac{c}{d} \Leftrightarrow ad = bc$

$\Leftrightarrow$ = if And only if (iff)

Solution:

$\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{b}(bd) = \frac{c}{d}(bd)$

$\Rightarrow \frac{a \cdot 1}{b}(bd) = \frac{c \cdot 1}{d}(bd)$

$\Rightarrow a \cdot \left(\frac{1}{b} \cdot b\right) \cdot d = c \cdot \left(\frac{1}{d} \cdot bd\right)$

$= c\left(bd \cdot \frac{1}{d}\right)$

$\Rightarrow ad = cb$

$\therefore ad = bc$

Again $ad = bc \Rightarrow (ad) \times \frac{1}{b} \cdot \frac{1}{d} = b \cdot c \cdot \frac{1}{b} \cdot \frac{1}{d}$

$\Rightarrow a \cdot \frac{1}{b} \cdot d \cdot \frac{1}{d} = b \cdot \frac{1}{b} \cdot c \cdot \frac{1}{d} \Rightarrow \frac{a}{b} = \frac{c}{d}$

Lecture No. 18

Exercise 1.1

Exercise 1.1

Q: No. 2 Do yourself

Q: No. 3 Do yourself

Q: No. 4:

i) $\frac{a}{c} + \frac{b}{c} = \frac{a+b}{c}$

Sol

Take L.H.S

$$\frac{a}{c} + \frac{b}{c} = a \times \frac{1}{c} + b \times \frac{1}{c}$$

$$= (a+b) \times \frac{1}{c}$$

$$= \frac{a+b}{c}$$

R.H.S

ii)

$$\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$$

Sol

Do yourself

Q: No. 5

$$-\frac{7}{12} - \frac{5}{18} = \frac{-21-10}{36}$$

Sol

Take

L.H.S $-\frac{7}{12} - \frac{5}{18}$

$$= -\frac{7}{12} \times 1 - \frac{5}{18} \times 1$$

$$= \frac{-7}{12} \times \left(3 \times \frac{1}{3}\right) - \frac{5}{18} \times \left(2 \times \frac{1}{2}\right)$$

$$= -\frac{21}{36} - \frac{10}{36}$$

$$= -21 \times \frac{1}{36} - 10 \times \frac{1}{36}$$

$$= (-21-10) \times \frac{1}{36}$$

$= -21 - 10 \times \frac{1}{36}$

$= \frac{-21-10}{36} = R.H.S$

Q: No. 6

(iii)

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}}$$

Sol

$$\frac{\frac{a}{b}+\frac{c}{d}}{\frac{a}{b}-\frac{c}{d}} = \frac{\frac{a}{b}\times\left(d\times\frac{1}{d}\right)+\left(b\times\frac{1}{b}\right)\times\frac{c}{d}}{\frac{a}{b}\times\left(d\times\frac{1}{d}\right)-\left(b\times\frac{1}{b}\right)\times\frac{c}{d}}$$

$$= \frac{\frac{a}{b}\times\frac{d}{d}+\frac{b}{b}\times\frac{c}{d}}{\frac{a}{b}\times\frac{d}{d}-\frac{b}{b}\times\frac{c}{d}} = \frac{\frac{ad}{bd}+\frac{bc}{bd}}{\frac{ad}{bd}-\frac{bc}{bd}}$$

$$= \frac{ad\times\frac{1}{bd}+bc\times\frac{1}{bd}}{ad\times\frac{1}{bd}-bc\times\frac{1}{bd}} = \frac{(ad+bc)\times\frac{1}{bd}}{(ad-bc)\times\frac{1}{bd}}$$

$$= \frac{(ad+bc) \times \frac{1}{bd}}{(ad-bc) \times \frac{1}{bd}}$$

concellation low

$$= \frac{ad+bc}{ad-bc}$$ R.H.S

Solve:

i) $\frac{4+16x}{4}$ Do your self

ii) $\frac{\frac{1}{4}+\frac{1}{5}}{\frac{1}{4}-\frac{1}{5}}$ Do yourself

iv) $\frac{\frac{1}{a}-\frac{1}{b}}{1-\frac{1}{a}\cdot\frac{1}{b}}$ Do yourself

Lecture No. 19

Exercise 1.2

Exercise 1.2

Q: No. 4

iv) $(-1)^{-\frac{21}{2}}$

$\because \sqrt{-1} = i$

Sol:

$$(-1)^{-\frac{21}{2}}$$

$$= \frac{1}{(-1)^{+\frac{21}{2}}} = \frac{1}{(-1)^{\frac{1}{2}\times 21}} = \frac{1}{\left((-1)^{1/2}\right)^{21}}$$

$$= \frac{1}{(\sqrt{-1})^{21}} = \frac{1}{(i)^{21}} = \frac{1}{i^{20}\cdot i}$$

$$= \frac{1}{i^{2\times 10}\cdot i} = \frac{1}{(i^2)^{10}\cdot i}$$

$$= \frac{1}{(-1)^{10}\cdot i} = \frac{1}{1\cdot i} = \frac{1}{i}$$

$$= \frac{1}{i}\times\frac{i}{i} = \frac{i}{i^2} = \frac{i}{-1} = -i$$

ii), i), iii) Do yourself

Q: No. 5

Sol: Do yourself

Q: No. 6

$(7, 9) + (3, -5) =$

Sol:

$(7, 9) + (3, -5) = (7+3,\ 9+(-5))$

$= (10,\ 9-5)$

$= (10, 4)$

Q: 7

Sol: Do yourself

Q: 12

$(5, -4) \div (-3, -8)$

Sol:

$(5, -4) \div (-3, -8)$

(11)

Since

$a^2 - b^2 = (a+b)(a-b)$

$(5,-4) \div (-3,-8)$

$= \dfrac{5,-4}{-3,-8} = \dfrac{5-4i}{-3-8i} \times \dfrac{-3+8i}{-3+8i}$

$= \dfrac{(5-4i)(-3+8i)}{(-3-8i)(-3+8i)}$

$= \dfrac{-15+40i+12i-32i^2}{(-3)^2-(8i)^2}$

$i^2 = -1$

$= \dfrac{-15+52i-32(-1)}{9-64i^2}$

$= \dfrac{-15+52i+32}{9-64(-1)}$

$= \dfrac{17+52i}{9+64} = \dfrac{17+52i}{73}$

$= \dfrac{17}{73} + \dfrac{52}{73}i$

Lecture No. 20

Exercise 1.2

Q: 13:

prove that the sum as well as product of any two conjugate complex number is a real number:

Sol

Let the two conjugate complex number be

$z = x+iy$ And $\bar{z} = x-iy$, where

$x, y \in R$

Sum:

$\text{Sum} = z+\bar{z} = x+iy+x-iy = 2x$

$= 2x \in R$

product:

$\text{product} = z \cdot \bar{z} = (x+iy)(x-iy)$

$= x \times x - x \times (iy) + iy \times x - (iy)(iy)$

$= x^2 - \cancel{ixy} + \cancel{ixy} - i^2 y^2$

$= x^2 - i^2 y^2 = x^2 - (-1) y^2 = x^2 + y^2$

$= x^2 + y^2 \in R, \quad x \in R, y \in R$

this is required result.

Q: 14

Find the multiple inverse

i) $(-4, 7)$

Sol

Let

$z = (-4, 7)$

multiple inverse of $z = \frac{1}{z}$

$$= \frac{1}{-4,7} = \frac{1}{-4+7i} \times \frac{-4-7i}{-4-7i}$$

$$= \frac{1 \times (-4-7i)}{(-4+7i)(-4-7i)}$$

$$= \frac{-4-7i}{(-4) \times (-4) + 4 \times 7i + 7i \times (-4) + 7i \times (-7i)}$$

$$= \frac{-4-7i}{16 + \cancel{28i} - \cancel{28i} - 49i^2}$$

$$= \frac{-4-7i}{16-49(-1)} = \frac{-4-7i}{16+49} \quad i^2 = -1$$

$= \dfrac{-4-7i}{16+49}$ Since $i^2 = -1$

$= \dfrac{-4-7i}{65} = \dfrac{-4}{65} - \dfrac{7}{65}i$

$= \left(\dfrac{-4}{65}, \dfrac{-7}{65}\right)$ Ans

Similarly solve all other part of Q: No 14 Do yourself

Q: No. 15

Factorize the following

ii) $9a^2+16b^2$

Sol

$-1 = i^2$

$9a^2+16b^2$

$= 9a^2-(-1)16b^2$

$= (3a)^2 - i^2 16b^2$

$= (3a)^2 - i^2(4b)^2$

$= (3a)^2 - (4bi)^2$

$$= (3a)^2 - (4bi)^2 \qquad \because a^2 - b^2 = (a+b)(a-b)$$

$$= (3a + 4bi)(3a - 4bi)$$

Ans

$$a^2 - b^2 = (a+b)(a-b)$$

Similarly

solve other part

Q: No. 15

Do yourself

Q: No 16

Separate into real and imaginary part the following

i) $\dfrac{2-7i}{4+5i}$

Sol

$$\frac{2-7i}{4+5i}$$

$$\frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{2-7i}{4+5i} \times \frac{4-5i}{4-5i}$$

$$= \frac{(2-7i) \times (4-5i)}{(4+5i)(4-5i)}$$

$$= \frac{(2-7i)(4-5i)}{(4)^2 - (5i)^2}$$

$$= \frac{2 \times 4 - 2 \times (5i) - 7i \times (4) - (7i) \times (-5i)}{16 - 25i^2}$$

$$= \frac{8 - 10i - 28i + 35i^2}{16 - 25(-1)}$$

$$= \frac{8 - 38i + 35(-1)}{16 + 25} = \frac{8 - 38i - 35}{41}$$

$$= \frac{-27 - 38i}{41}$$

$$= \frac{-27}{41} - \frac{38}{41}i$$

Ans

ii)

$$\frac{(-2+3i)^2}{1+i}$$

we know that

$$(a+b)^2 = a^2+b^2+2ab$$

Sol

$$\frac{(-2+3i)^2}{1+i} = \frac{(-2)^2+(3i)^2+2(-2)(3i)}{1+i}$$

$$= \frac{4+9(-1)-12i}{1+i} = \frac{4-9-12i}{1+i}$$

$$= \frac{-5-12i}{1+i} \times \frac{1-i}{1-i}$$

$$= \frac{(-5-12i)\times(1-i)}{(1+i)(1-i)}$$

complete this Question Do Yourself

iii) $\frac{i}{1+i}$ Do Yourself

Lecture No. 21

Arithmetic Mean

and

Arithmetic Mean

For Grouped and Ungrouped Data

Arithmetic Mean

The sum of all the values and numbers of observation.

Let $x_1, x_2, x_3, \dots x_n$

$$A.M = \frac{\text{Sum of all values}}{\text{number of all values}}$$

$$A.M = \frac{\sum_{i=1}^{n} x_n}{n}$$

Exp:

5, 4, 9, 11, 12

$$A.M = \frac{\text{Sum of all values}}{\text{number of values}}$$

$$A.M = \frac{5+4+9+11+12}{5}$$

$$A.M = \frac{41}{5}$$

$$A.M = 8.2$$

Q-NO. 1 Find Arithmetic mean

85, 60, 70, 35, 45, 55, 60

Q. NO. 2 Find Arithmetic Mean

101, 105, 67, 98, 70

Q. NO. 3 Find Arithmetic Mean

205, 110, 250, 108, 250

Q. NO. 4 Find Arithmetic Mean

111, 130, 85, 95, 66, 39

Arithmetic mean for Ungrouped Data :-

Formula:-

$$\bar{X} = \frac{\text{Sum of all observation}}{\text{No of observation}}$$

$$\bar{x} = \frac{\Sigma x}{n}$$

Example :-

Find Arithmetic mean of following ungrouped data.

12, 56, 31, 22, 25, 39

$$\text{Arithmetic mean} = \bar{x} = \frac{\Sigma x}{n}$$

$$\bar{x} = \frac{12+56+31+22+25+39}{6}$$

$$\bar{x} = \frac{185}{6}$$

$$\bar{x} = 30.83$$

Q-NO-1 Arithmetic Mean for ungrouped Data.

12, 30, 15, 17, 20, 26, 17, 40

Q-NO-2 Arithmetic mean for Ungrouped Data.

13, 14, 16, 27, 31, 50, 60, 44

Q-NO-3 Find Arithmetic Mean for ungrouped Data.

60, 70, 80, 90, 30, 40, 50

Lecture No. 22

Arithmetic Mean For Grouped Data

Arithmetic Mean for Grouped Data

Formula:

$$\bar{X} = \frac{\Sigma f x}{\Sigma f}$$

Example: Find Arithmetic mean for grouped Data.

3, 3, 4, 5, 5, 5, 3, 3

x	f	fx
3	2	3×2=6
4	1	4×1=4
5	3	5×3=15
3	2	3×2=6

$$\bar{X} = \frac{\Sigma f x}{\Sigma f}$$

$$= \frac{31}{8}$$

$$\bar{X} = 3.875$$

Q-NO-1 Find Arithmetic Mean for grouped Data.

3, 6, 2, 6, 5, 3, 2, 6, 3

Q-NO-2 Find Arithmetic Mean for grouped Data.

60, 20, 60, 50, 20, 50, 60

Q-NO-3 Find Arithmetic mean for grouped Data.

13, 12, 15, 15, 12, 13, 15, 12

Lecture No. 23

Exercise

Sol (i)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A + C = C + A$$

$$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

Take L.H.S

$$A + C = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-1 & 2+0 & 3+0 \\ 2+0 & 3-2 & 1+3 \\ 1+1 & -1+1 & 0+2 \end{bmatrix} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \rightarrow (1)$$

Now
Take R.H.S

$$C + A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$$

$$C + A = \begin{bmatrix} -1+1 & 0+2 & 0+3 \\ 0+2 & -2+3 & 3+1 \\ 1+1 & 1-1 & 2+0 \end{bmatrix}$$

$$C + A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 1 & 4 \\ 2 & 0 & 2 \end{bmatrix} \rightarrow (11)$$

From Eq (1) & (11)
L.H.S = R.H.S

Hence prove
$A + C = C + A$

(93) $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$

Solve (x)

$2A + 2B = 2(A+B)$

Take L.H.S

$$2A+2B = 2\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 6 \\ 4 & 6 & 2 \\ 2 & -2 & 0 \end{bmatrix} + \begin{bmatrix} 2 & -2 & 2 \\ 4 & -4 & 4 \\ 6 & 2 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 2+2 & 4-2 & 6+2 \\ 4+4 & 6-4 & 2+4 \\ 2+6 & -2+2 & 0+6 \end{bmatrix} = \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \rightarrow (I)$$

Now:

Take R.H.S

$$2(A+B) = 2\left(\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix} + \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}\right)$$

$$= 2\begin{bmatrix} 1+1 & 2-1 & 3+1 \\ 2+2 & 3-2 & 1+2 \\ 1+3 & -1+1 & 0+3 \end{bmatrix} = 2\begin{bmatrix} 2 & 1 & 4 \\ 4 & 1 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 2 & 8 \\ 8 & 2 & 6 \\ 8 & 0 & 6 \end{bmatrix} \rightarrow (II)$$

From Eq (I) & (II)

L.H.S = R.H.S

Sol
=

(iii)

$A+A^t$ is Symmetric

Since

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$$

Now

$$A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}^t$$

$$A^t = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

(i) if A is Symmetric then

$$(A)^t = A$$

$$A^t = A$$

(ii) if A is Skew Symmetric then

$$A^t = -A$$

And

$$A+A^t = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+0 \\ 0+2 & 1+1 \end{bmatrix}$$

$$A+A^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} \rightarrow ①$$

If $A+A^t$ is Symmetric then

$$(A+A^t)^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}^t = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = A+A^t \text{ by } ①$$

$$(A+A^t)^t = A+A^t$$

So prove $A+A^t$ is Symmetric

Sol

iv) $A - A^t$ is skew symmetric

If $A - A^t$ is skew then prove

for skew

$$(A - A^t)^t = -(A - A^t)$$

Similarly prove all other

results

Hence prove (96)

$2A+2B=2(A+B)$

Similarly prove other results

Q:

If $A=\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $B=\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

then verify that

i) $(A+B)^t = A^t + B^t$ ii) $(A-B)^t = A^t - B^t$

iii) $A+A^t$ is Symmetric

iv) $A-A^t$ is skew Symmetric

v) $B+B^t$ is Symmetric

vi) $B-B^t$ is skew Symmetric

Sol (i) $(A+B)^t = A^t + B^t$

Take L.H.S

$(A+B)^t$

Now: $A+B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$

(97)

$$A+B=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}$$

$$A+B=\begin{bmatrix}1+1 & 2+1\\ 0+2 & 1+0\end{bmatrix}=\begin{bmatrix}2 & 3\\ 2 & 1\end{bmatrix}$$

Now

$$(A+B)^t=\begin{bmatrix}2 & 3\\ 2 & 1\end{bmatrix}^t$$

$$(A+B)^t=\begin{bmatrix}2 & 2\\ 3 & 1\end{bmatrix}\rightarrow (1)$$

Now
Take R.H.S

$$A^t+B^t=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}^t+\begin{bmatrix}1 & 1\\ 2 & 0\end{bmatrix}^t$$

$$A^t+B^t=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\ 1 & 0\end{bmatrix}$$

$$A^t+B^t=\begin{bmatrix}1+1 & 0+2\\ 2+1 & 1+0\end{bmatrix}=\begin{bmatrix}2 & 2\\ 3 & 1\end{bmatrix}\rightarrow (11)$$

See from Eq (1) & (11)

L.H.S = R.H.S

Hence prove
$(A+B)^t=A^t+B^t$

Lecture No. 24

Exercise

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1+1 & 2+1 \\ 3+1 & 1+0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Similarly solve other results

Q:2

If $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \end{bmatrix}$; then find

i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ ii) $B + \begin{bmatrix} -2 \\ 3 \end{bmatrix}$

iii) $C + \begin{bmatrix} -2 & 1 & 3 \end{bmatrix}$ iv) $D + \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$

v) $2A$ vi) $(-1)B$ vii) $(-2)C$

viii) $3D$ ix) $3C$

Solve the Following Question:

Q. 1:

i) $\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$.

ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \left(\begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\right)$

iii) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & -1 & -1 \\ 0 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$

iv) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 3 & 1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 & -2 \\ -2 & -1 & 0 \\ 0 & 2 & -1 \end{bmatrix}$

v) $\left(\begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

Sol (i)

$$\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ 3 & 0 \end{bmatrix}\right) + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0 & 0+2 \\ 0+3 & 1+0 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$= \begin{bmatrix} 1+1 & 2 \end{bmatrix}$

Q: 3

(100)

If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 & 1 \\ 2 & -2 & 2 \\ 3 & 1 & 3 \end{bmatrix}$

$C = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -2 & 3 \\ 1 & 1 & 2 \end{bmatrix}$

verify the following Results:

i) $A + C = C + A$ ii) $A + B = B + A$

iii) $B + C = C + B$ iv) $A + (B + A) = 2A + B$

v) $(C - B) + A = C + (A - B)$

vi) $2A + B = A + (A + B)$

vii) $(C - B) - A = (C - A) - B$

viii) $(A + B) + C = A + (B + C)$

ix) $A + (B - C) = (A - C) + B$

x) $2A + 2B = 2(A + B)$

Sol

Since $A = \begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix}$

i) $A + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$$\begin{bmatrix} -1 & 2 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1+1 & 2+1 \\ 2+1 & 1+1 \end{bmatrix} = \begin{bmatrix} 0 & 3 \\ 3 & 2 \end{bmatrix}$$

ix) $C = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$

$3C$

$$3C = 3\begin{bmatrix} 1 & -1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3\times 1 & -1\times 3 & 3\times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -3 & 6 \end{bmatrix}$$

Similarly Solve other Results

Lecture No. 25

Contineous and Dis-Contineous Function

contineous-function:

A function $f(x)$ is said to be contineous at $x = a$ if

i) $f(a)$ is define

ii) $\lim_{x \to a} f(x)$ exists.

iii) $\lim_{x \to a} f(x) = f(a)$

oR

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$$

Example:

Discuss the continuity of the function

$$f(x) = 3x^2 - 5x + 4 \quad \text{at} \quad x = 1$$

Sol

$f(x) = 3x^2 - 5x + 4$

At $x=1$ So $f(x)$ become

$f(1) = 3(1)^2 - 5(1) + 4 = 3 - 5 + 4$

$f(1) = 2 \rightarrow$ (I)

Now:

$\lim_{x \to 1} f(x) = \lim_{x \to 1} (3x^2 - 5x + 4)$

$= 3(1)^2 - 5(1) + 4 = 3 - 5 + 4$

$\lim_{x \to 1} f(x) = 2 \rightarrow$ (II)

As by (I) & (II)

$\lim_{x \to 1} f(x) = f(1)$

So

$f(x)$ is continuous at

$x = 1$

$\Rightarrow f(1)$ is not defined

Thus $f(x)$ is discontineous at $x=1$

Note:

i) $\lim\limits_{x \to a^-} f(x) =$ Left hand limit (L.H.L)

ii) $\lim\limits_{x \to a^+} f(x) =$ Right hand limit (R.H.L)

For contineous Function

$$\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x)$$

For Discontineous Function

$$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+}$$

Discontineous Function:

A function $f(x)$ is said to be discontineous at $x = a$ if

$$\lim_{x \to a} f(x) \neq f(a)$$

Example

Discuss the continuity of the function

$$f(x) = \frac{x^2 - 1}{x - 1} \quad \text{at} \quad x = 1$$

Sol

AS

$$f(x) = \frac{x^2 - 1}{x - 1}$$

At $x = 1$ so $f(x)$ become

$$f(1) = \frac{(1)^2 - 1}{1 - 1} = \frac{1 - 1}{1 - 1} = \frac{0}{0}$$

Q:

Discuss continuity of the following

i)

$$f(x) = 2x^2 + x - 5, \text{ at } x = 1$$

ii)

$$f(x) = \frac{x^2 - 9}{x - 3} \quad \text{at } x = -3$$

Sol

(ii)

$$f(x) = \frac{x^2-9}{x-3}$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$L.H.L = \lim_{x \to (-3)^-} f(x)$$

$$= \lim_{x \to -3^-} \frac{x^2-9}{x-3} = \lim_{x \to -3^-} \frac{x^2-(3)^2}{x-3}$$

$$= \lim_{x \to -3^-} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}} = \lim_{x \to -3^-} (x+3)$$

$$= -3+3 = 0$$

$\rightarrow$ ①

R.H.L

$$= \lim_{x \to -3^+} f(x)$$

$$= \lim_{x \to -3^+} \frac{x^2-9}{(x-3)}$$

$$= \lim_{x \to -3^+} \frac{\cancel{(x-3)}(x+3)}{\cancel{(x-3)}}$$

$$= \lim_{x \to -3^+} (x+3)$$

$$= -3+3 = 0 \quad \rightarrow (ii)$$

Now

$$f(x) = \frac{x^2-9}{x-3}$$

$$\lim_{x \to -3} f(x) = \lim_{x \to -3} \frac{x^2-9}{x-3}$$

$$= \frac{(-3)^2-9}{-3-3} = \frac{(3)^2-9}{-6}$$

$$= \frac{9-9}{-6} = \frac{0}{-6} = 0 \quad \rightarrow (iii)$$

From (i), (ii) & (iii)

R.H.L = R.H.L

So function is continuous

Do yourself (i)

Lecture No. 26

Binomial Theorem and Characteristic

Binomial theorem:

If a And b are two real number and n is positive integer then

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots + {}^nC_n a^{n-n} b^n$$

The above formula is called Binomial theorm

OR

If a And b are two real numbers And n is positive integer then

$$(a+b)^n = \binom{n}{0} a^n + \binom{n}{1} a^{n-1} b^1 + \binom{n}{2} a^{n-2} b^2 + \binom{n}{3} a^{n-3} b^3 + \cdots + \binom{n}{n} a^{n-n} b^n$$

the above formula is called Binomial theorm

Since:

we know that

$(a+b)^2 = a^2 + b^2 + 2ab \rightarrow (i)$

Now:

using binomial theorem

to verify Eq (i)

Since:

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots + {}^nC_n a^{n-n} b^n \longrightarrow (A)$$

Now:

put $n = 2$ in Eq (A)

$$(a+b)^2 = a^2 + {}^2C_1 a^{2-1} b + {}^2C_2 a^{2-2} b^2$$

$$(a+b)^2 = a^2 + {}^2C_1 ab + {}^2C_2 a^0 b^2$$

$${}^2C_1 = 2$$

$$(a+b)^2 = a^2 + 2ab + b^2$$

$${}^2C_2 = 1$$

$$a^0 = 1$$

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$a^0 = 1$$

Lecture No. 27

Exercise Binomial

Exercine 8.2

Q: 1

Using Binomial theorm Expend the following

i) $(a+2b)^5$

Sol

We know that

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + {}^nC_3 a^{n-3} b^3 + \cdots + {}^nC_n a^{n-n} b^n$$

Now. $n = 5$

$$(a+2b)^5 = a^5 + {}^5C_1 a^{5-1}(2b) + {}^5C_2 a^{5-2}(2b)^2 + {}^5C_3 a^{5-3}(2b)^3 + {}^5C_4 a^{5-4}(2b)^4 + {}^5C_5 a^{5-5}(2b)^5$$

$$= a^5 + 5a^4(2b) + (10)a^3(4b^2) + 10a^2(8b^3) + 5a^1(16)b^4 + (1)a^0(32)b^5$$

$$= a^5 + 10a^4 b + 40a^3 b^2 + 80a^2 b^3 + 80ab^4 + 32b^5$$

$\because a^0 = 1$

ii)

(iii) Expand 6 terms

$$\left(\frac{x}{2}-\frac{2}{x^2}\right)^6$$

Since:

$$(a+b)^n = a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \cdots$$

$n=6$

Sol

$$\left(\frac{x}{2}-\frac{2}{x^2}\right)^6 = \left(\frac{x}{2}\right)^6 + {}^6C_1\left(\frac{x}{2}\right)^{6-1}\left(\frac{-2}{x^2}\right) + {}^6C_2\left(\frac{x}{2}\right)^{6-2}\left(-\frac{2}{x^2}\right)^2$$

$$+ {}^6C_3\left(\frac{x}{2}\right)^{6-3}\left(\frac{-2}{x^2}\right)^3 + {}^6C_4\left(\frac{x}{2}\right)^{6-4}\left(-\frac{2}{x^2}\right)^4$$

$$+ {}^6C_5\left(\frac{x}{2}\right)^{6-5}\left(-\frac{2}{x^2}\right)^5 + {}^6C_6\left(\frac{x}{2}\right)^{6-6}\left(-\frac{2}{x^2}\right)^6$$

$$= \frac{x^6}{64} - {}^6C_1\left(\frac{x}{2}\right)^5\left(\frac{2}{x^2}\right) + {}^6C_2\left(\frac{x}{2}\right)^4\left(\frac{2}{x^2}\right)^2$$

$$+ {}^6C_3\left(\frac{x}{2}\right)^3\left(\frac{-2}{x^2}\right)^3 + {}^6C_4\left(\frac{x}{2}\right)^2\left(\frac{2}{x^2}\right)^4$$

$$- {}^6C_5\left(\frac{x}{2}\right)^1\left(\frac{2}{x^2}\right)^5 + {}^6C_6\left(\frac{x}{2}\right)\left(\frac{2}{x^2}\right)^6$$

$$= \frac{x^6}{64} - \frac{3x^2}{8} + \frac{15}{4} - \frac{20}{x^3} + \frac{60}{x^6} - \frac{96}{x^9} + \frac{64}{x^{12}}$$

Q: 2

calculate the Following by means of binomial theorem:

i) $(0.97)^3$ $\qquad$ $0.97 = 1 - 0.03$

Sol

$$(0.97)^3 = (1-0.03)^3$$

$$= (1)^3 + {}^3C_1 (1)^{3-1}(-0.03) + {}^3C_2 (1)^{3-2} (-0.03)^2 + {}^3C_3 (1)^{3-3} (-0.03)^3$$

$$= 1 + 3(1)^2(-0.03) + 3(1)^1(0.0009) + (1)(-0.000024)$$

$$= 1 - 0.09 + 0.0027 - 0.000027$$

$$= 0.912673$$

(113)

iii Do yourself

iv) Do yourself

v) Do yourself

vi) Do yourself

Q. No. 1

Q: 2

all other part solve
Do yourself

Lecture No. 28

Trigonometric Function

Trigonometric Function:

Sin x, cos x, Tan x, cosecant x,

Secant x, cot x

or

Sine, cose, Tangent, cosecant

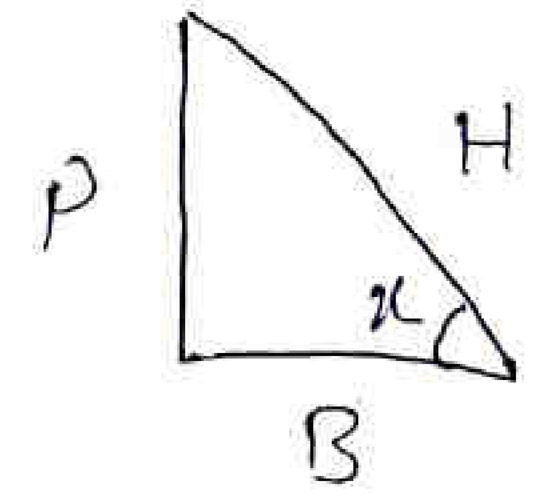
Secant, cot x

are Trigonometric function:

Now:

Define Trigonometric Function:

consider triangle



$$\sin x = \frac{P}{H}$$

$$\cos x = \frac{B}{H}$$

$$\tan x = \frac{P}{B}$$

P = perpendicular

H = Hypotenuse

B = Base

$$\text{cosecant } x\ (\csc x) = \frac{H}{P}$$

Secant x (sec x) = $\frac{H}{B}$ (115)

$\cot x = \frac{B}{P}$

i) $\sin^2 x + \cos^2 x = 1$

ii) $1 + \tan^2 x = \sec^2 x$

iii) $1 + \cot^2 x = \operatorname{cosec}^2 x$

iv) $\sec^2 x - 1 = \tan^2 x$

v) $\sin^2 x = 1 - \cos^2 x$

vi) $\cos^2 x = 1 - \sin^2 x$

vii) $\tan x = \frac{\sin x}{\cos x}$

viii $\cot x = \frac{\cos x}{\sin x}$

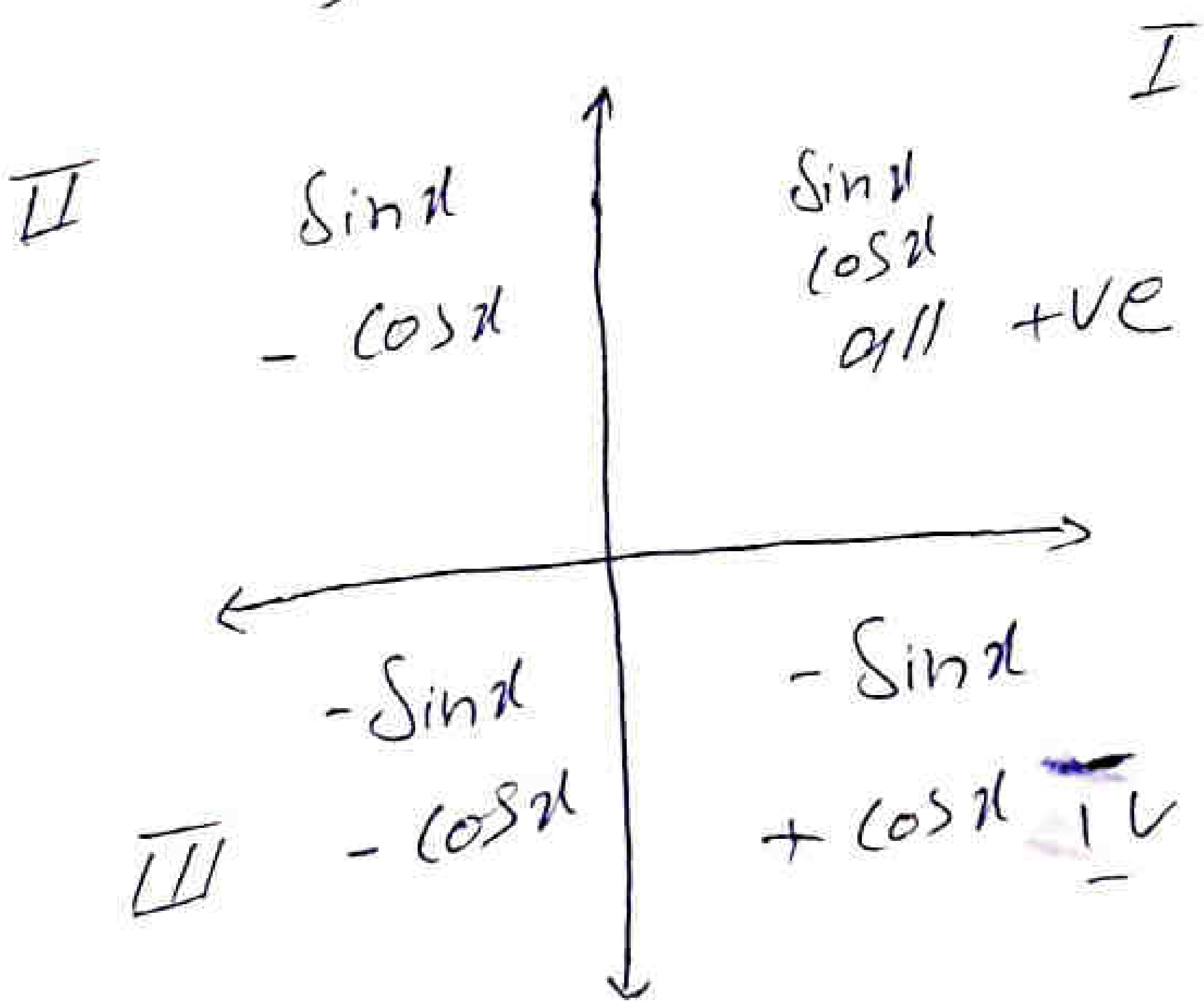
ix) $\sec x = \frac{1}{\cos x}$

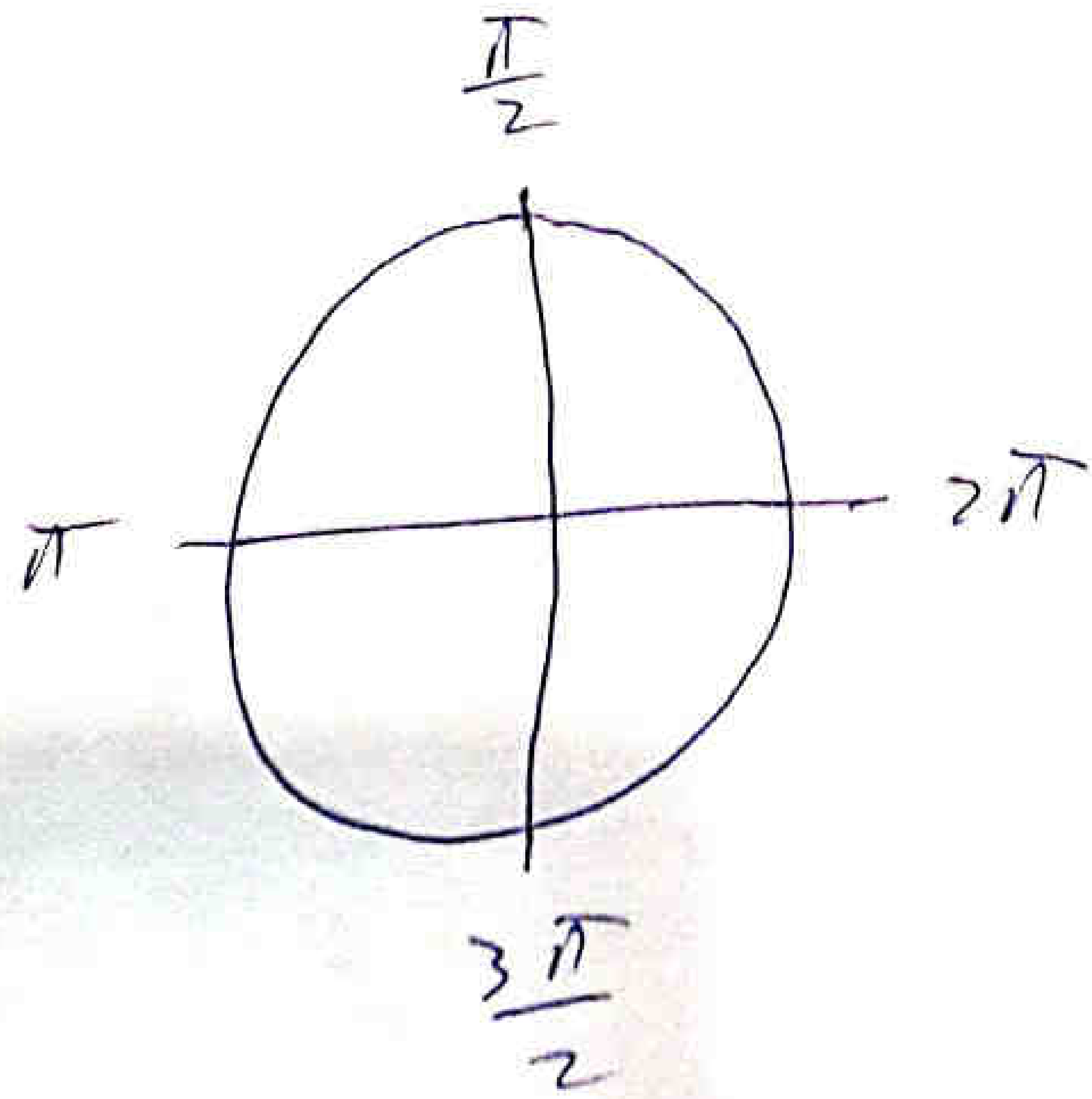
x) $\operatorname{cosec} x = \frac{1}{\sin x}$

value of Trigonometric Function:

Total Angle of circle is 2π

Since $\pi = 180^\circ$, $2\pi = 2(180) = 360$





(117)

Lecture No. 29

Derivation

Derivative:-

The derivative of f(x) with respect to x is the function f'(x) and is defined as,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Note:- Derivative means change/slope.

Representation of derivative are

$\frac{dy}{dx}$, $\frac{dy}{dt}$, y', $f'(x)$

Example:-

$$y = 5x^2 + x$$

$$\frac{d}{dx}(y) = \frac{d}{dx}(5x^2 + x)$$

$$\frac{dy}{dx} = 5\frac{d}{dx}x^2 + \frac{d}{dx}(x)$$

$$= 5(2x) + 1$$

$$\frac{dy}{dx} = 10x + 1$$

Q-NO-1

$$y = x^3 + x^2 + 1$$

$$\frac{d}{dx}(y) = \frac{d}{dx}(x^3 + x^2 + 1)$$

$$\frac{dy}{dx} = \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2) + \frac{d}{dx}(1)$$

$$= 3x^2 + 2x + 0$$

$$\frac{dy}{dx} = 3x^2 + 2x$$

Note:- Derivative of all constants is zero.

Q-NO-2 Solve

$$y = 5x^4 + x^3 + x^2$$

Q-NO-3 Solve

$$y = x^5 + 6x^2 + 8$$

Q-NO-4 Solve

$$y = 8x + 3x^2 + 9x^3$$

Differentiate with respect to x (w.r.t x)

i) $y = x^4 + 2x^3 + x^2$

ii) $y = 2x^{-3} + 2x^{-\frac{3}{2}} + 3$

iii) $y = x^{-19} + \frac{1}{x^5} + xy$

iv) $y = xy + 5y^2 + x^2 y + \frac{x^{10}}{y}$

Lecture No. 30

Integration

Integration
(Anti-derivative)

the reverse process of derivative is called integration or anti-derivative.

2nd-Defination of Integration:

A function F(x) is an antiderivative or integration of function f(x) if

$$F'(x) = f(x)$$

Sign of Integration

$\int$ or $\int$

Mathematicall form

$$\int f(x)\,dx = F(x)$$

Example:

(x^2+x)

Derivation

$\frac{d}{dx}(x^2+x)$

$= 2x^{2-1}+1$

$= \frac{d}{dx}(x^2+x)$

$= \frac{d}{dx}(x^2) + \frac{d}{dx}(x)$

$= 2x^{2-1}+1$

$= 2x^1+1$

$= 2x+1$

Integration

$\int (2x+1)\,dx$

$= \int 2x\,dx + \int 1\,dx$

$= 2\int x\,dx + 1\int dx$

$= 2 \cdot \frac{x^2}{2} + \int 1\,dx$

$= \cancel{2} \cdot \frac{x^2}{\cancel{2}} + x$

$= x^2 + x$

Find Integration of $x^5 + x^2$

Sol

=

$\int (x^5+x^2)\,dx = \int x^5\,dx + \int x^2\,dx$

$= \frac{x^{5+1}}{5+1} + \frac{x^{2+1}}{2+1} = \frac{x^6}{6} + \frac{x^3}{3}$

Q:

(124)

Find Integration

i) $2x + x^2$

ii) $2x + xy + 5$

iii) $3x^2 + 6x + 3$

iv) $\sin x$, $\cos x$

v) $3x + 5$

vi) $xy + y^2$

Sol

v)

$3x+5$

$$\int (3x+5)\,dx = \int 3x\,dx + \int 5\,dx$$

$$= 3\int x\,dx + 5\int dx$$

$$= 3\cdot\frac{x^2}{2} + 5\cdot x$$

$$= \frac{3x^2}{2} + 5x$$

vi)

$xy + y^2$ w.r.t x

Sol $\int (xy + y^2)\,dx$

$$= \int xy\,dx + \int y^2\,dx$$

$$= y\int x\,dx + y^2\int 1\,dx$$

$$= y\frac{x^2}{2} + y^2 x$$

Do yourself next part

Assignment No. 1

ⓐ

Assignments:

If

$A = \begin{bmatrix} 3 & 2 \\ 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 4 \\ -3 & -5 \end{bmatrix}$

then verify that

i) $(AB)^t = B^t A^t$

ii) $(AB)^{-1} = B^{-1} A^{-1}$

*

If $A = \begin{bmatrix} 0 & 1 \\ 2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 4 \\ 5 & -2 \end{bmatrix}$

then prove that

$AB \neq BA$

*

(b)

Find the value of a, b, c & d the Following matrix

$$\begin{bmatrix} a+c & a+2b \\ c-1 & 4d-6 \end{bmatrix} = \begin{bmatrix} 0 & -7 \\ 3 & 2d \end{bmatrix}$$

*

verify that if

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$$

then

i) $(A^t)^t = A$

ii) $(B^t)^t = B$

Assignment No. 2

©

ASSignment:

*

verify De-Morgan's laws if

$U = \{1, 2, 3 \ldots 20\}$,

$A = \{2, 4, 6, \ldots, 20\}$, $B = \{1, 3, 5, \ldots 19\}$

*

If $A = \{2, 4, 6, \ldots, 20\}$, $B = \{1, 3, 5, \ldots 19\}$

then verify that

i) $A \cap (A \cup B) = A \cup (A \cap B)$

ii) $A \cup (A \cap B) = A \cap (A \cup B)$

)

1

* verify that:

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$$

* Integrate

i) $\int \left(xy + 5x + 5y + \frac{y}{x}\right) dx$

ii) $\int \left(5x^{-3} + \frac{x}{y} + 6x + \frac{8}{x}\right) dx$

* derivate the following

i) $\frac{d}{dx}\left(xy + 5x + 5y + \frac{y}{x}\right)$

ii) $\frac{d}{dx}\left(5x^{-3} + \frac{x}{y} + 6x + \frac{8}{x}\right)$